Algebra and Trigonometry

Stewart

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Chapter 0 Prerequisites

0.2 Real Numbers

1. Real Numbers Introduction
2. Properties of Real Numbers

$\vdots$

6. Sets and Intervals

0.2.6 Sets and Intervals

Questions 41 – 66

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 4, 6, 8\}$

$C = \{7, 8, 9, 10\}$

41. (a) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(b) $A \cap B = \{2, 4, 6\}$

42. (a) $B \cup C = \{2, 4, 6, 7, 8, 9, 10\}$

(b) $B \cap C = \{8\}$

43. (a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap C = \{7\}$

44. (a) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(b) $A \cap B \cap C = \emptyset$

$A = \{x \mid x \geq -2\}$ $B = \{x \mid x < 4\}$

$C = \{x \mid -1 < x \leq 5\}$

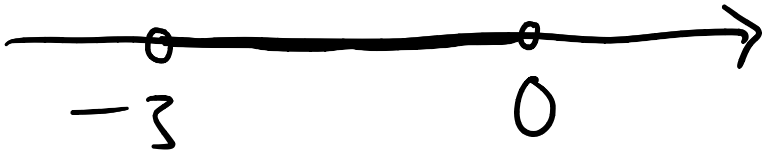
45. (a) $B \cup C = \{x \mid x \leq 5\}$

(b) $B \cap C = \{x \mid -1 < x < 4\}$

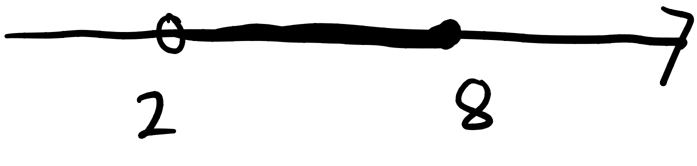
46. (a) $A \cap C = \{x \mid -1 < x \leq 5\}$

(b) $A \cap B = \{x \mid -2 \leq x < 4\}$

47. $(-3, 0) = \{x \mid -3 < x < 0\}$

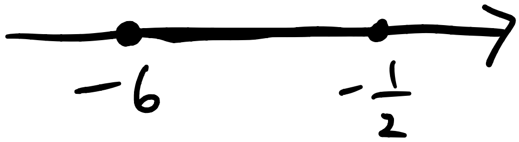


48. $(2, 8] = \{x \mid -2 < x \leq 8\}$



49. $[2, 8) = \{x \mid 2 \leq x < 8\}$

50. $[-6, -\frac{1}{2}] = \{x \mid -6 \leq x \leq -\frac{1}{2}\}$



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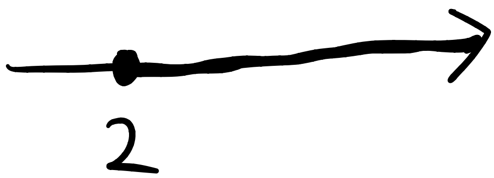
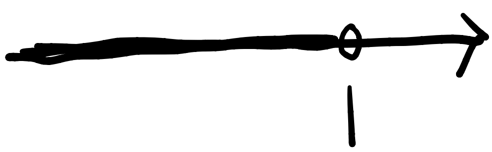
51. $[2, \infty) = \{x \mid x \geq 2\}$



52. $(-\infty, 1) = \{x \mid x < 1\}$



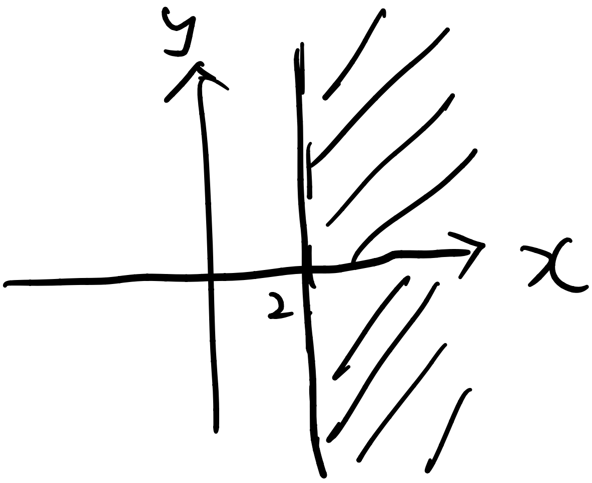
53. $x \leq 1 = x \in (-\infty, 1]$

54. $1 \leq x \leq 2 = x \in [1, 2]$

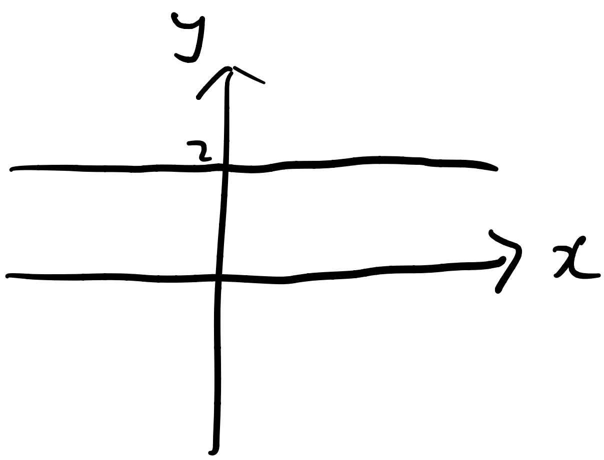
Chapter 1 Equations and Graphs

1.1 The Coordinate Plane

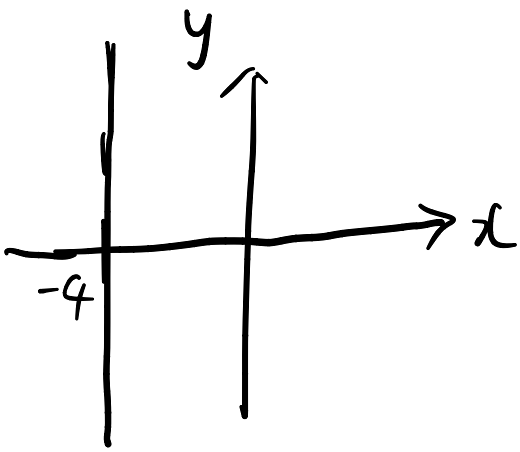
9. $\{(x,y) \mid x \geq 2\}$



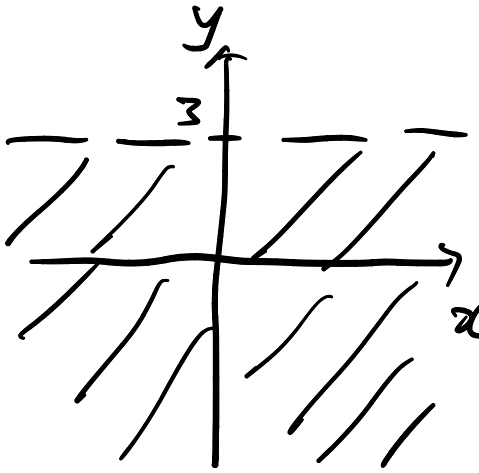
10. $\{(x,y) \mid y = 2\}$



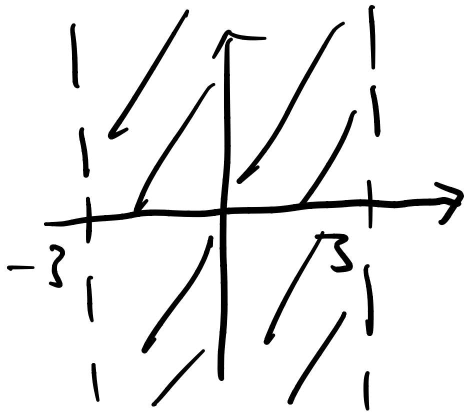
11. $\{(x,y) \mid x = -4\}$



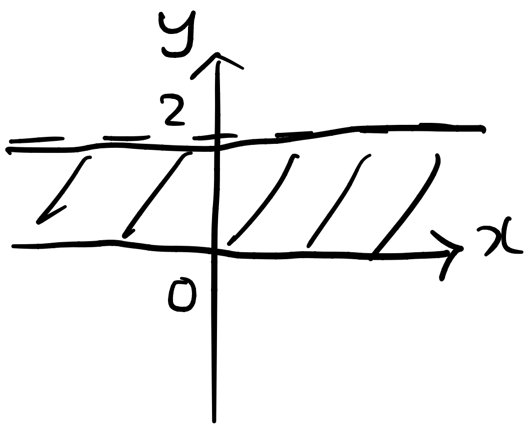
12. $\{(x,y) \mid y < 3\}$



13. $\{(x,y) \mid -3 < x < 3\}$



14. $\{(x,y) \mid 0 \leq y \leq 2\}$



Distance and Midpoint

21. $P1 = (0, 2)$

$P2 = (3, 0)$

$$\text{Distance} = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{2^2 + (-3)^2}$$

$$= \sqrt{13}$$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{0 + 3}{2}, \frac{2 + 0}{2} \right)$$

$$= \left(\frac{3}{2}, 1 \right)$$

1.1 The Coordinate Plane

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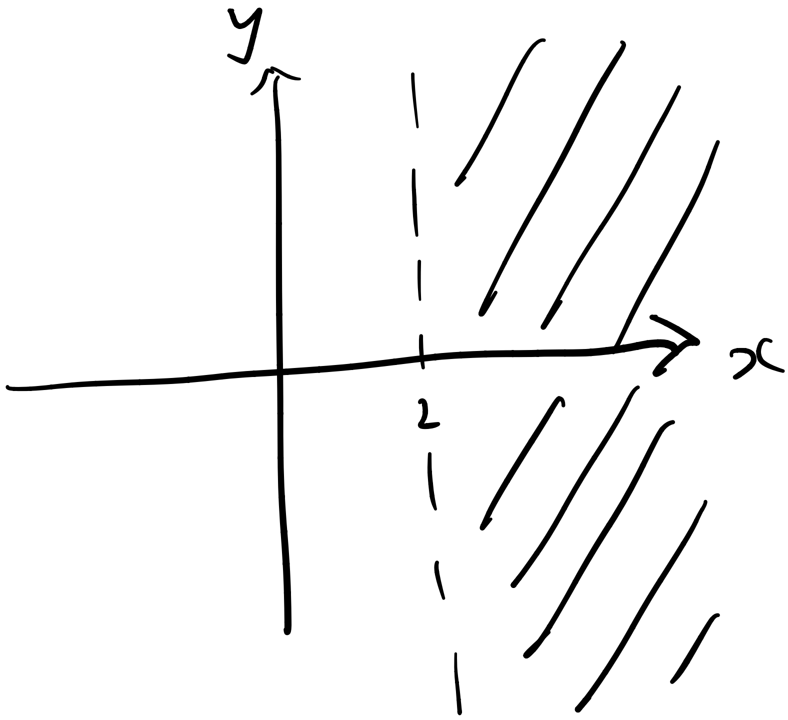
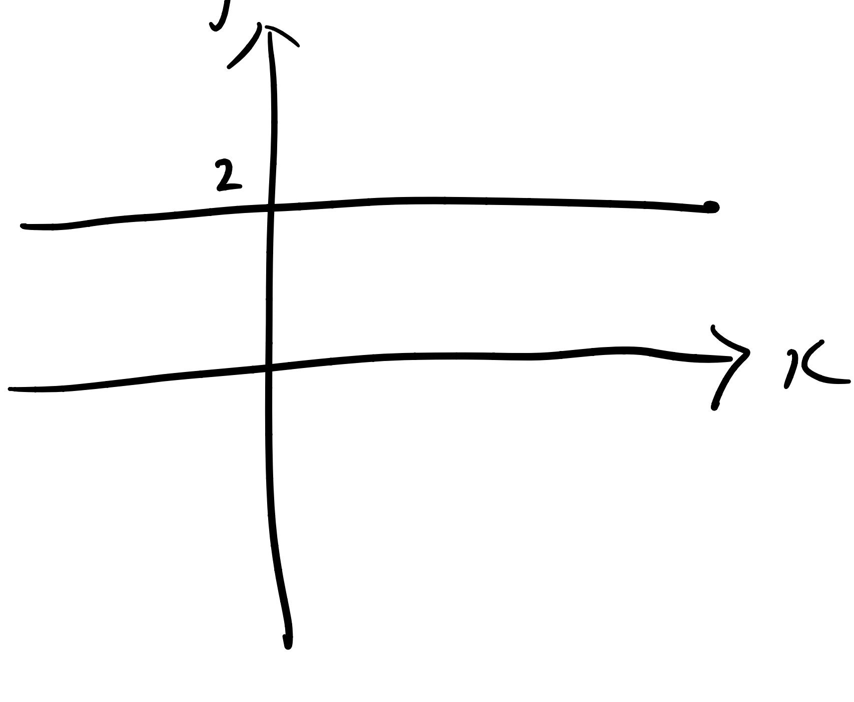
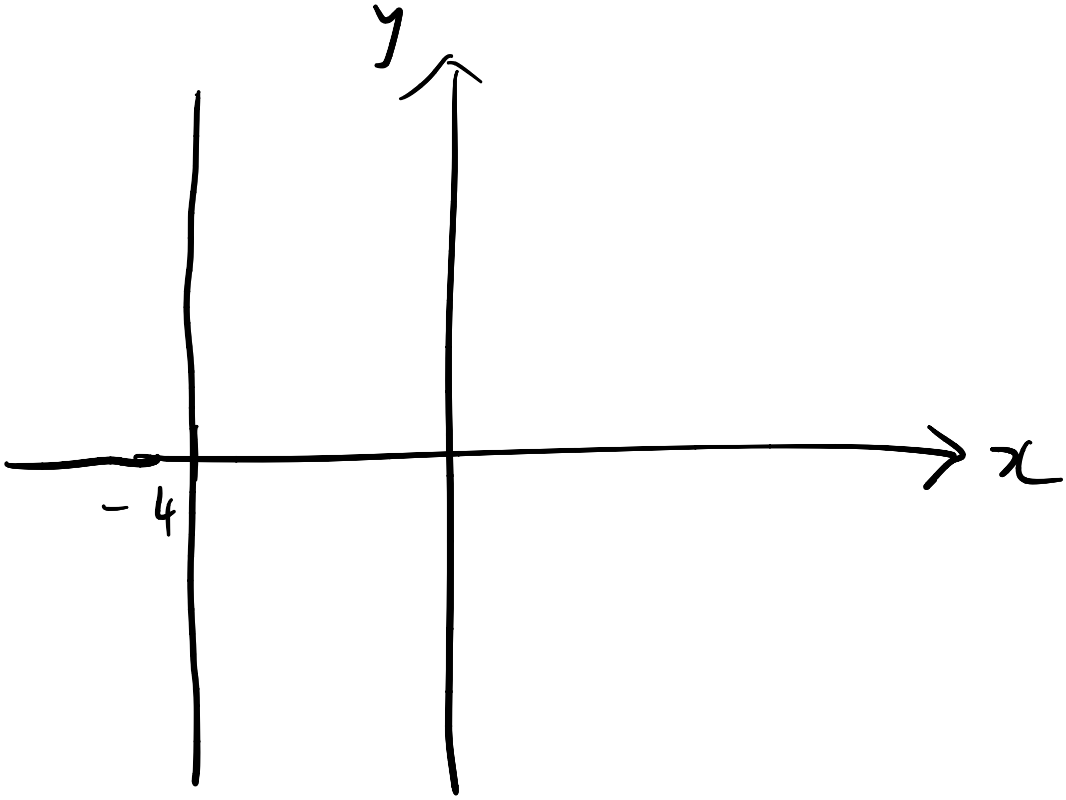
9. $\{(x,y) \mid x \geq 2\}$



10. $\{(x,y) \mid y=2\}$



11. $\{(x, y) \mid x = -4\}$



1.2 Graphs of Equations in Two Variables; Circles

Graphing Equations by Plotting Points

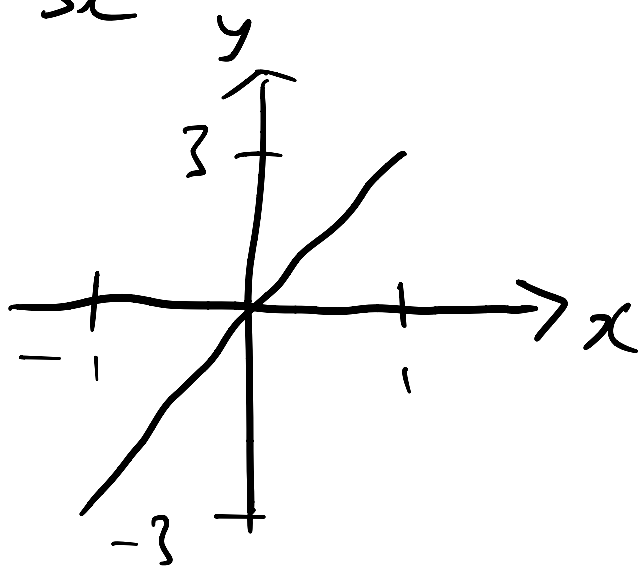
Intercepts

Circles

Symmetry

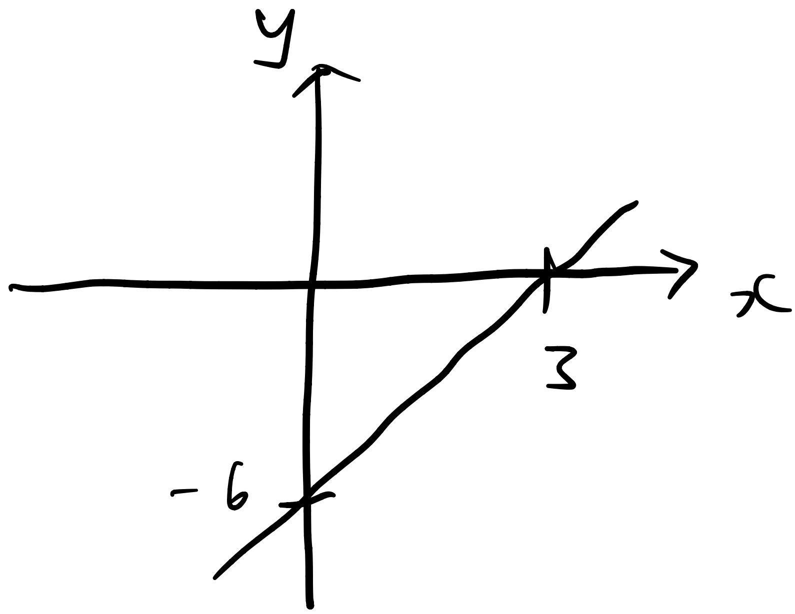
Graphing Equations

15. $y = 3x$



19. $2x - y = 6$

$y = 2x - 6$



49. $y = x^2 - 5$ Intercepts

x-intercept: $0 = x^2 - 5$

$x^2 = 5$

$x = \pm\sqrt{5}$

y-intercept: $y = 0 - 5$

$y = -5$

55. $4x^2 + 25y^2 = 100$

x-intercept: $4x^2 + 25(0) = 100$

$4x^2 = 100$

$x^2 = 25$

$x = \pm 5$

y-intercept: $4(0) + 25y^2 = 100$

$25y^2 = 100$

$y^2 = 4$

$y = \pm 2$

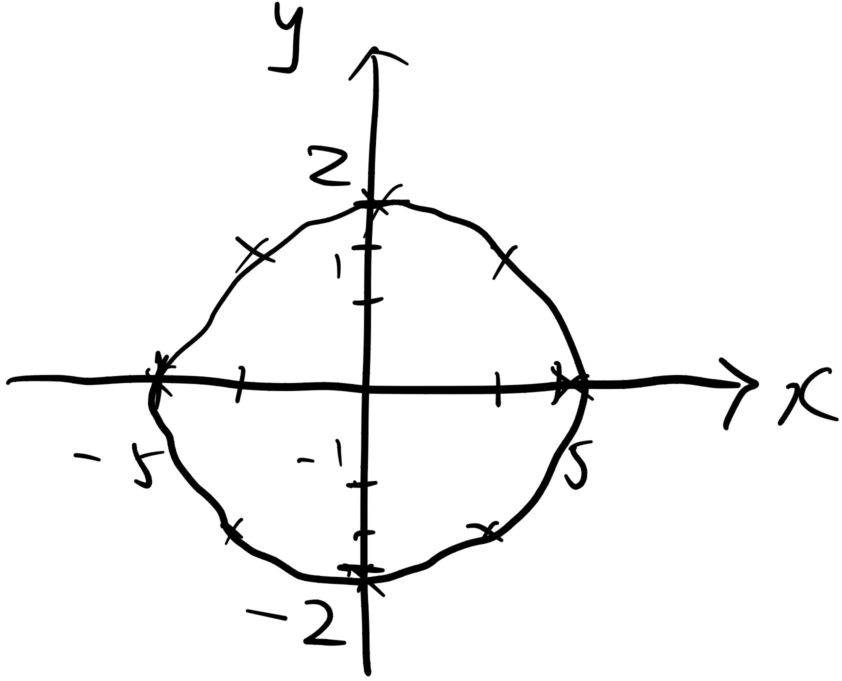
$x = 3$, $4(9) + 25y^2 = 100$

$25y^2 = 64$

$y^2 = \frac{64}{25}$

$y = \pm \frac{8}{5}$

$x = -3$, $y = \pm \frac{8}{5}$

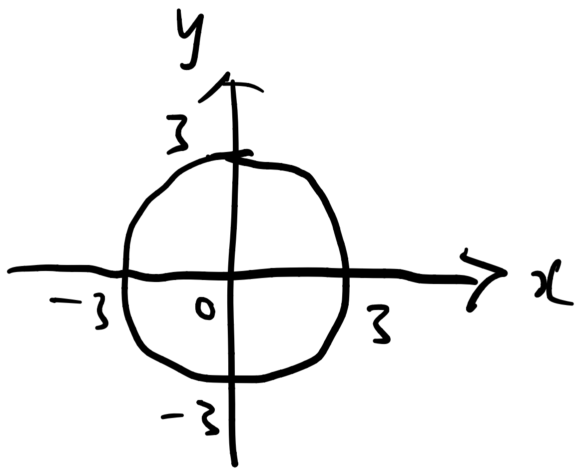


y
2
1
x
-5
-1
5
-2

Graphing Circles

67. $x^2+y^2=9$

$x^2+y^2=3^2$

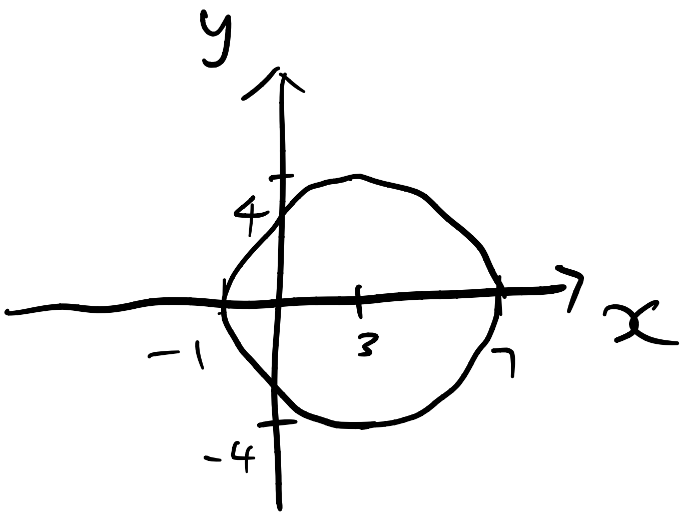


69. $(x-3)^2+y^2=16$

$(x-3)^2+y^2=4^2$

when $y=0$, $x=-1, 7$

when $x=3$, $y=-4, 4$



73. $C(-3, 2)$, $r = 5$

$$(-3+3)^2 + (2-2)^2 = 5^2$$

$$\therefore (x+3)^2 + (y-2)^2 = 5^2$$

Symmetry

95. $y = x^4 + x^2$

if symmetric, $(x, y) = (-x, -y)$

$y = (-x)^4 + (-x)^2$

$-y = x^4 + x^2$

Eq 1 $\neq$ Eq 2

$\therefore$ not symmetric w.r.t. the origin

97. $y = x^3 + 10x$

$(-y) = (-x)^3 + 10(-x)$

$-y = -x^3 - 10x$

$y = x^3 + 10x$

$\because$ Eq 1 $=$ Eq 2

$\therefore$ Symmetric w.r.t. the origin

99. $x^4y^4+x^2y^2=1$

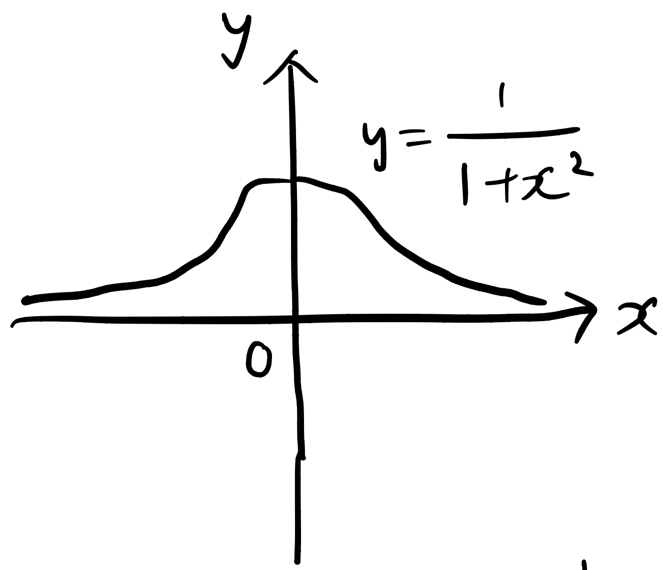
$(-x)^4(-y)^4+(-x)^2(-y)^2=1$

$x^4y^4+x^2y^2=1$

Eq 1 = Eq 2

$\therefore$ Symmetric w.r.t. the origin

101.



symmetric with respect to y-axis

$(x, y) = (-x, y)$

1.2 Graphs of Equations in Two Variables; Circles

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1. $2, 3$

$$2y = x + 1$$

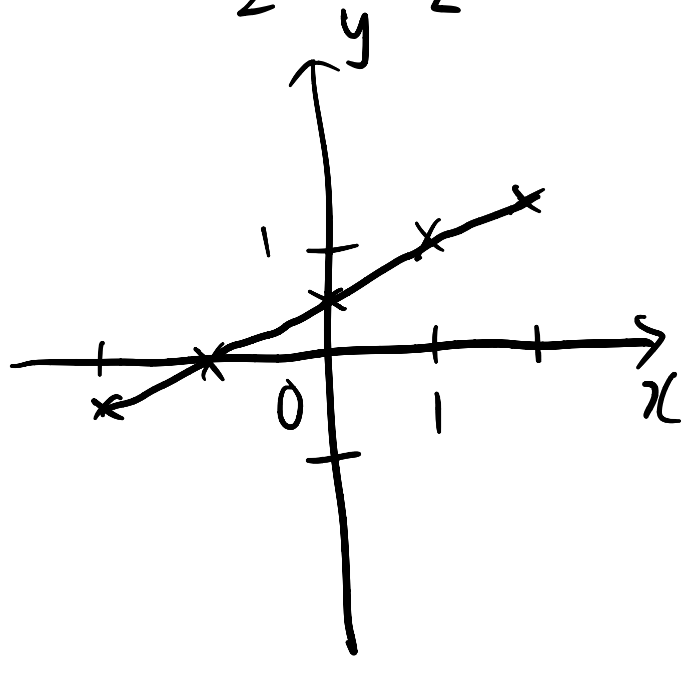
$$2(3) = (2) + 1$$

$$6 \neq 3$$

$\therefore$ No the point is not on the graph.

$$y = \frac{1}{2}x + \frac{1}{2}$$

x	y	(x, y)
-2	$-\frac{1}{2}$	$(-2, -\frac{1}{2})$
-1	0	$(-1, 0)$
0	$\frac{1}{2}$	$(0, \frac{1}{2})$
1	1	$(1, 1)$
2	$\frac{3}{2}$	$(2, \frac{3}{2})$



9. $y = 3 - 4x$

$(0, 3)$, $3 = 3 - 4(0)$

$3 = 3$

$\therefore$ Yes

$(4, 0)$, $0 = 3 - 4(4)$

$0 \neq -13$

$\therefore$ No

$(1, -1)$, $-1 = 3 - 4(1)$

$-1 = -1$

$\therefore$ Yes

1.3 Lines

9. $P(-1, 2)$, $Q(0,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-2}{0-(-1)} = \frac{-2}{1} = -2$$

25. $P(2,3)$, $m = 5$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 5(x-2)$$

$$y - 3 = 5x - 10$$

$$y - 5x + 7 = 0$$

$$5x - y - 7 = 0$$

29. $P_1(2,1)$, $P_2(1,6)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6-1}{1-2}$$

$$= \frac{5}{-1}$$

$$= -5$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -5(x-2)$$

$$-5x + 10 - y + 1 = 0$$

$$5x + y - 11 = 0$$

23. $m = 3$, $c = -2$

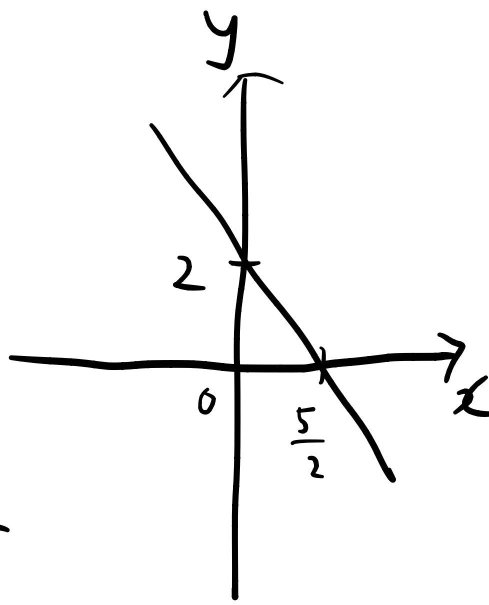
$$y = 3x - 2$$

61. $4x + 5y = 10$

$$5y = -4x + 10$$

$$y = -\frac{4}{5}x + 2$$

$\therefore$ slope $= -\frac{4}{5}$, y-intercept $= 2$



Vertical and Horizontal Lines

35. $P(1, 3)$, $m = 0$

$$y = mx + c$$
$$= 0 + c$$
$$= c$$

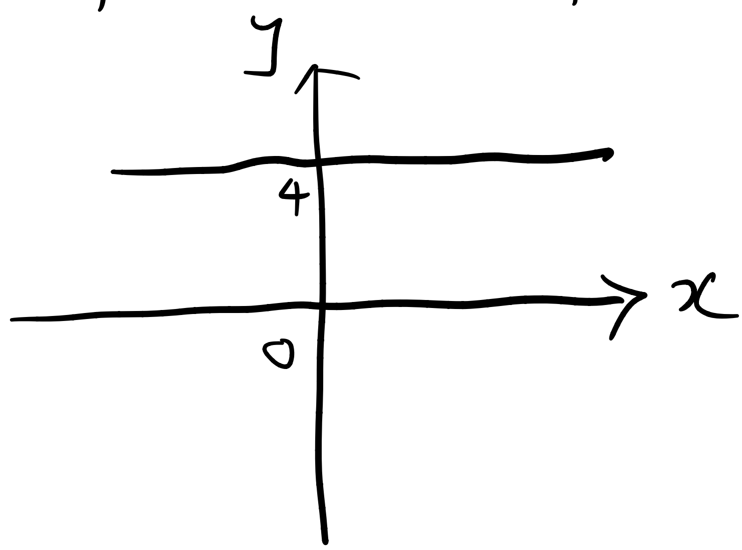
$$y = 3$$

37. $P(2, -1)$, m: undefined

$$x = 2$$

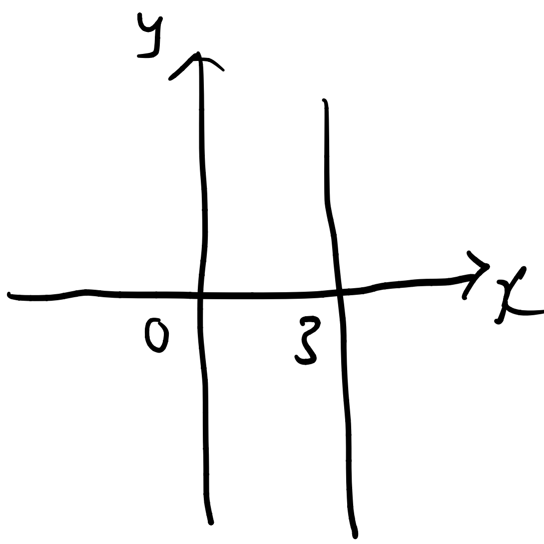
63. $y = 4$

$\therefore$ Slope $= 0$, y-intercept $= 4$



65. $x = 3$

$\therefore$ Slope $=$ undefined, no y-intercept



67. $5x+2y-10=0$

x-intercept: $5x+2(0)-10=0$

$$5x=10$$

$$x=2$$

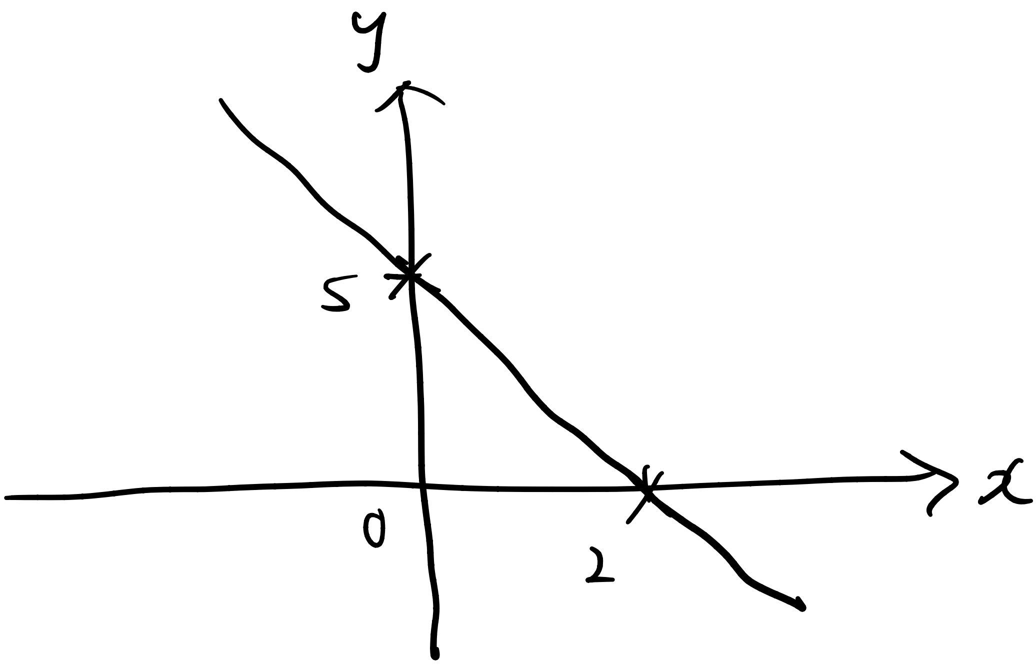
$P_1\ (2,0)$

y-intercept: $5(0)+2y-10=0$

$$2y=10$$

$$y=5$$

$P_2\ (0,5)$



43. P(1, −6), parallel to the line
$x+2y=6$

$$x+2y=6$$
$$2y=-x+6$$
$$y=-\frac{1}{2}x+3$$

$$m=-\frac{1}{2}$$

$$y-y_1=m(x-x_1)$$
$$y-(-6)=-\frac{1}{2}(x-1)$$
$$y+6=-\frac{1}{2}x+\frac{1}{2}$$
$$\frac{1}{2}x+y+6-\frac{1}{2}=0$$
$$\therefore x+2y+11=0$$

81.

If line $AB \perp BC$, $AB \perp DA$,

$AB // CD$,

ABCD is a rectangle.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{11-1} = \frac{2}{10} = \frac{1}{5}$$

$$m_{DA} = \frac{6-1}{0-1} = \frac{5}{-1} = -5$$

$\because m_{AB} m_{DA} = -1$

$\therefore AB \perp DA$

$$m_{BC} = \frac{8-3}{10-11} = \frac{5}{-1} = -5$$

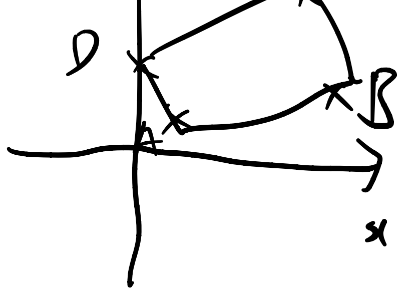
$\because m_{AB} m_{BC} = -1$

$\therefore AB \perp BC$

$$m_{CD} = \frac{6-8}{0-10} = \frac{-2}{-10} = \frac{1}{5}$$

$\because m_{AB} = m_{CD}$

$\therefore AB // CD$



$\therefore$ A, B, C and D are vertices of a rectangle.

47. $P(-1,-2)$, perpendicular to $2x+5y+8=0$

$$2x+5y+8=0$$

$$5y=-2x-8$$

$$y=-\frac{2}{5}x-\frac{8}{5}$$

$$m=-\frac{2}{5}$$

$$m_1m_2=-1$$

$$-\frac{2}{5}m_2=-1$$

$$m_2=\frac{5}{2}$$

$$y-y_1=m(x-x_1)$$

$$y-(-2)=\frac{5}{2}(x-(-1))$$

$$y+2=\frac{5}{2}(x+1)$$

$$\frac{5}{2}x+\frac{5}{2}-y-2=0$$

$$5x+5-2y-4=0$$

$$\therefore 5x-2y+1=0$$

53. $y = -2x + b$

$\therefore$ all the lines have the same slope -2

87. $T = 0.02t + 15.0$

(a) The slope 0.02 represents for every year that passes since 1950, there is a 0.02°C rise in surface temperature.

The T-intercept represents the surface temperature at the start of the first year : 1950 which is 15.0°C.

(b)
$$T = 0.02(2050 - 1950) + 15.0$$
$$= 0.02(100) + 15.0$$
$$= 2 + 15.0$$
$$= 17.0\,°C$$

1.6 Solving Other Types of Equations

Polynomial Equations

5. $x^2 - x = 0$

$x(x-1) = 0$

$\therefore x = 0, 1$

6. $3x^3 - 6x^2 = 0$

$3x^2(x-2) = 0$

$\therefore x = 0, 2$

Equations Involving Rational Expressions

27. $$\frac{1}{x-1}+\frac{1}{x+2}=\frac{5}{4}$$

$$\frac{x+2+x-1}{(x-1)(x+2)}=\frac{5}{4}$$

$$\frac{2x+1}{(x-1)(x+2)}-\frac{5}{4}=0$$

$$\frac{4(2x+1)-5(x-1)(x+2)}{4(x-1)(x+2)}=0$$

$$\frac{8x+4-5(x^2-x+2x-2)}{4(x-1)(x+2)}=0$$

Numerator:

$$=8x+4-5x^2-5x+10$$

$$=-5x^2+3x+16$$

$$4(x^2+x-2)$$

Chapter 2 Functions

Content

1. Functions
2. Graphs of Functions
3. Getting Information from the Graph of a Function
4. Average Rate of Change of a Function
5. Linear Functions and Models
6. Transformations of Function
7. Combining Functions
8. One-to-One Functions and Their Inverses

Example 1

$$f(x) = x^2 + 4$$

(a) x is squared and added to 4

(b) $f(3) = 9+4$
$= 13$

$f(-2) = 4+4$
$= 8$

$f(\sqrt{5})$
$= 5+4$
$= 9$

(c) domain : $x \in \mathbb{R}$, set of all real numbers

range : $\boxed{1}$ $\{y \mid y \geq 4\}$ / $[4, \infty)$

(d)

$$x \longrightarrow \boxed{f} \longrightarrow x^2+4$$

$\boxed{1}$

$$x \longrightarrow \boxed{\text{square then add } 4} \longrightarrow x^2+4$$

Example 7 Finding Domains of Functions 16/9/23

(a) $f(x) = \dfrac{1}{x^2 - x}$

$= \dfrac{1}{x(x-1)}$

Domain: $\{x \mid x \neq 0 \text{ and } x \neq 1\}$

(b) $g(x) = \sqrt{9 - x^2}$

$9 - x^2 \geq 0$

$x^2 - 9 \leq 0$

$(x+3)(x-3) \leq 0$

$-3 \leq x \leq 3$

Domain: $\{x \mid -3 \leq x \leq 3\}$ / $[-3, 3]$

(c) $h(t) = \dfrac{t}{\sqrt{t+1}}$

$t + 1 > 0$

$t > -1$

Domain: $\{t \mid t > -1\}$

/ $(-1, \infty)$

2.2.1 Graphing Functions by Plotting Points

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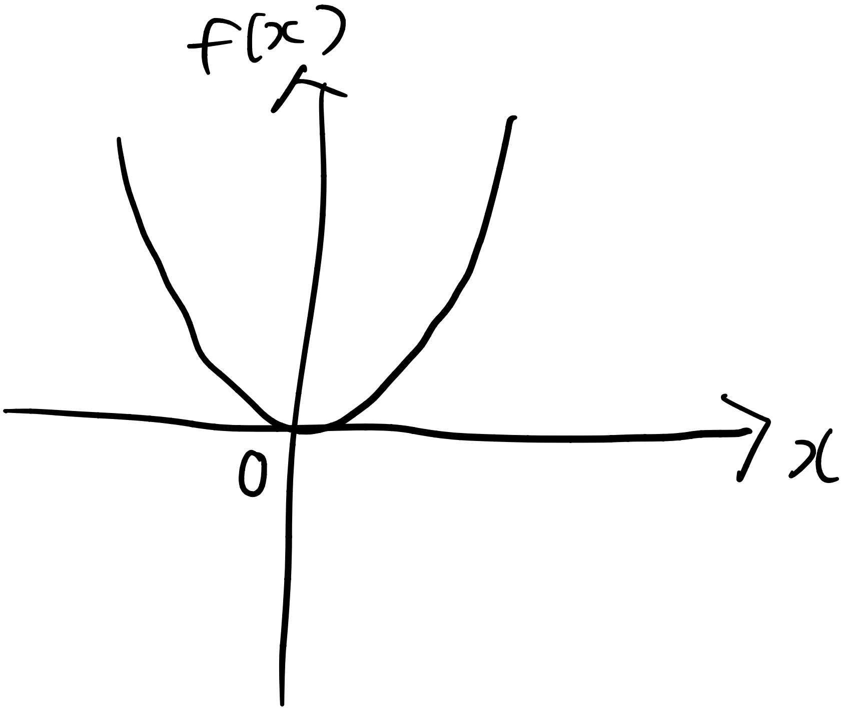
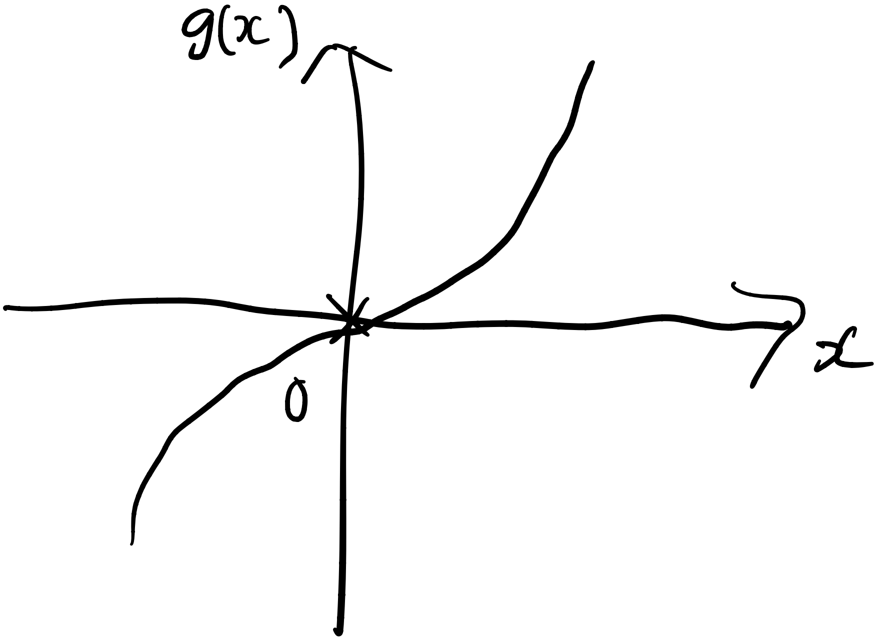
Example 1

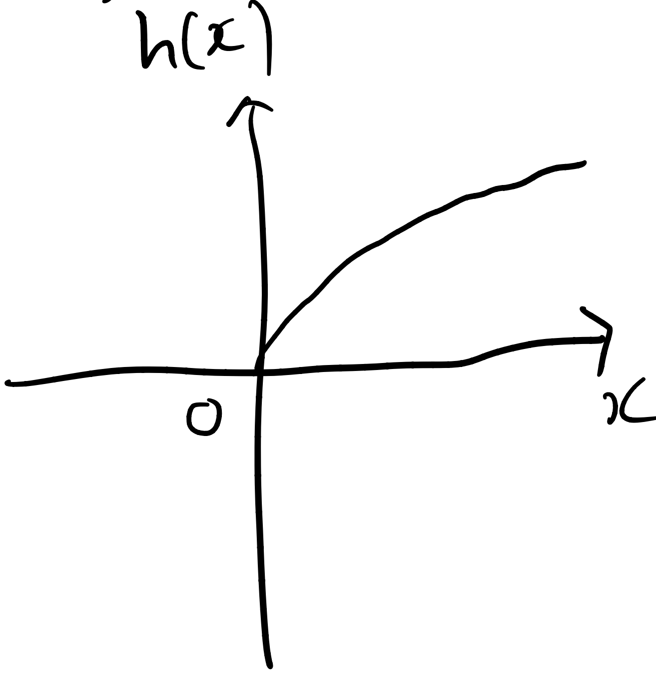
(a) $f(x) = x^2$



(b) $g(x) = x^3$



(c) $h(x) = \sqrt{x}$



2.2.3 Graphing Piecewise Defined Functions

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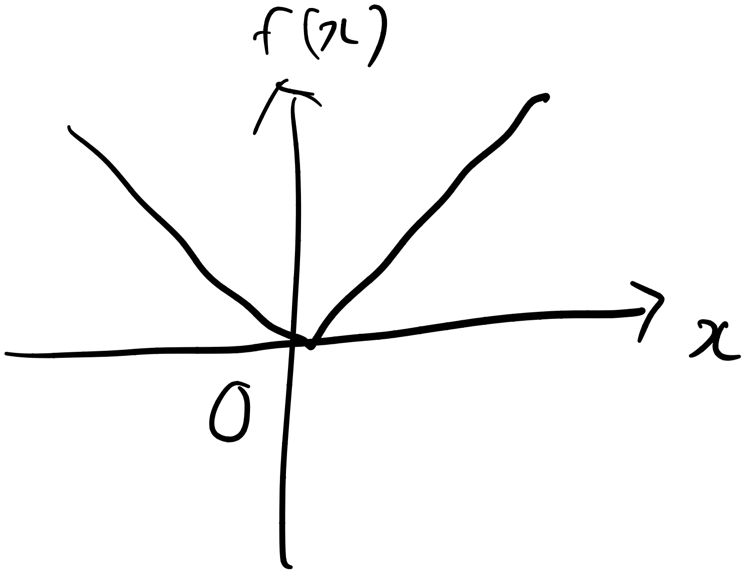
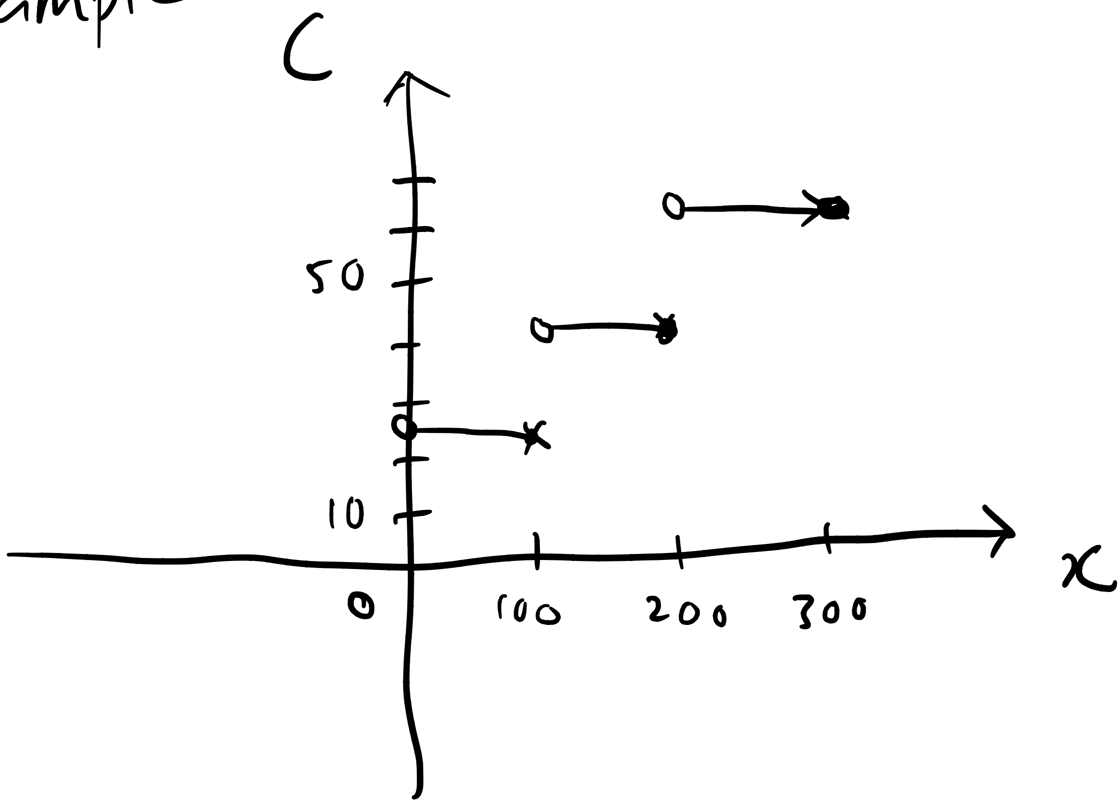
Example 5

$f(x) = |x|$



Example 7



2.3.1 Values of a Function; Domain and Range

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Example 1

(a) $T(1) = 25^\circ F$

$T(3) = 30^\circ F$

$T(5) = 20^\circ F$

(b) $T(2)$

(c) $x = 1, 4$

(d) $1 \leq x \leq 4$

(e) $T(3) - T(1)$

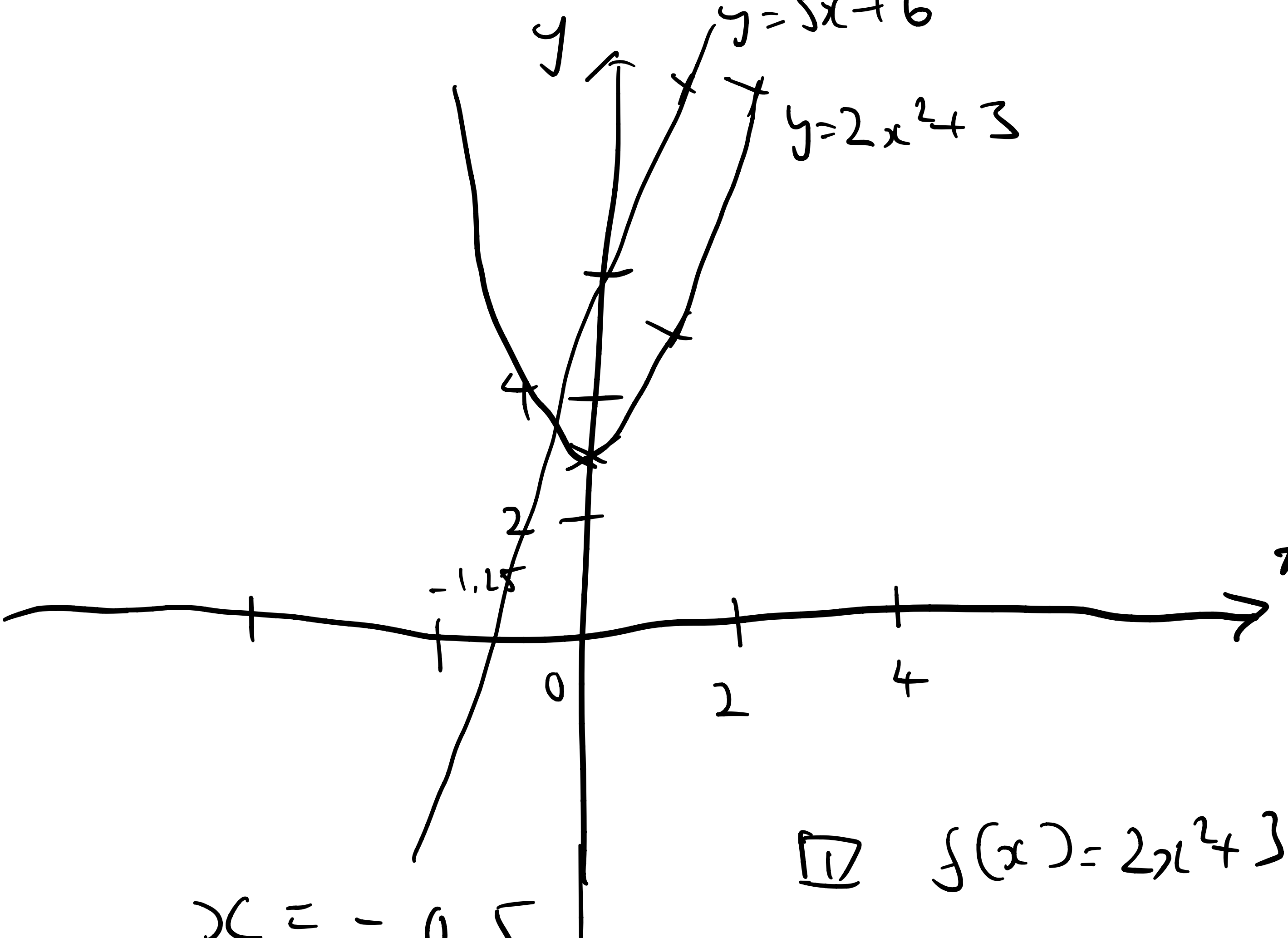
$= 30 - 25$

$= 5^\circ F$

2.3.2 Comparing Function Values: Solving Equations and Inequalities Graphically

Example 3

(a) $2x^2+3=5x+6$



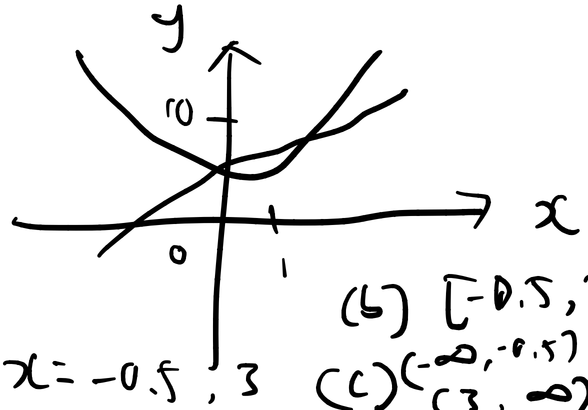
$x=-0.5$

(b) $x \geq 0.5$

(c) $x<-0.5$

$\boxed{II}$ $f(x)=2x^2+3$

$g(x)=5x+6$



(a) $x=-0.5$; 3

(b) $[-0.5, 3]$

(c) $(-\infty, -0.5) \cup (3, \infty)$

2.2 Exercises

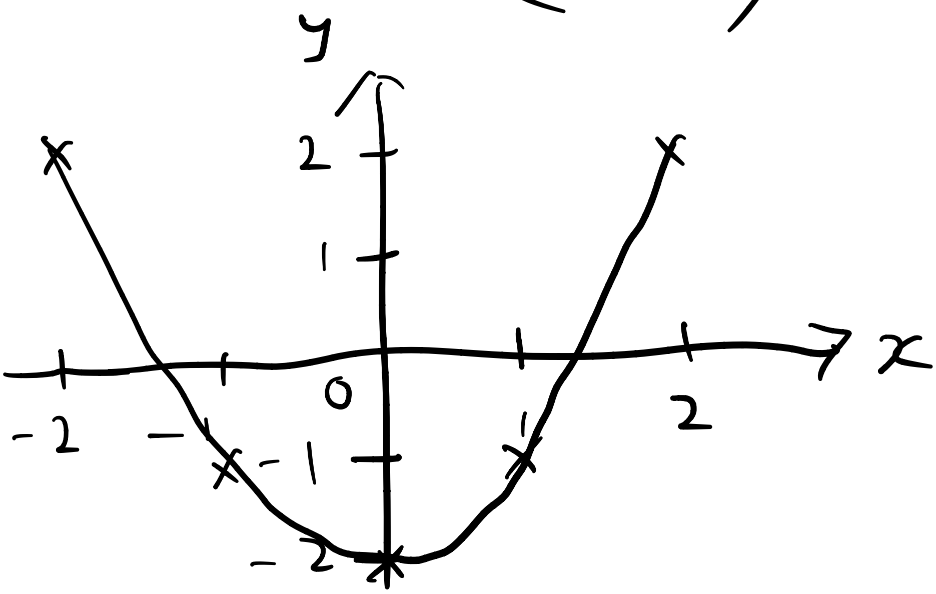
1. $(x, f(x))$

(x, x^2-2)

$(3, 7)$

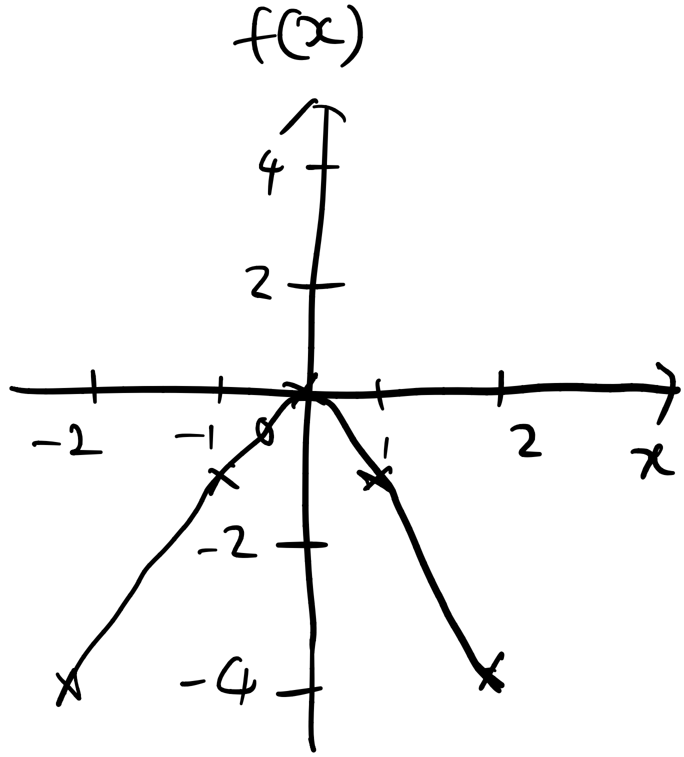
7

x	$f(x)$	(x, y)
-2	2	$(-2, 2)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
2	2	$(2, 2)$



9. $f(x) = -x^2$

x	$f(x)$
-2	-4
-1	-1
0	0
1	-1
2	-4



0.5 Algebraic Expressions

1. a, d and f

15. $(6x-3)+(3x+7)$

$= 9x+4$

37. $(3t-2)(7t-4)$

$= 3t(7t-4)-2(7t-4)$

$= 21t^2-12t-14t+8$

$= 21t^2-26t+8$

45. $(5x+1)^2 = 25x^2+10x+1$

67. $(x+2)(x^2+2x+3)$

$= x(x^2+2x+3)+2(x^2+2x+3)$

$= (x^3+2x^2+3x)+(2x^2+4x+6)$

$= x^3+4x^2+7x+6$

0.8 Solving Basic Equations

1. Equations Involving Fractional Expressions
2. Power Equations

① Equations Involving Fractional Expressions

49. $\frac{1}{z} - \frac{1}{2z} - \frac{1}{5z} = \frac{10}{z+1}$

$$\frac{10}{10z} - \frac{5}{10z} - \frac{2}{10z} = \frac{10}{z+1}$$

$$\frac{3}{10z} = \frac{10}{z+1}$$

$$3(z+1) = 10(10z)$$

$$3z + 3 = 100z$$

$$97z = 3$$

$$z = \frac{3}{97}$$

51. $\frac{x}{2x-4} - 2 = \frac{1}{x-2}$

$$\frac{x}{2x-4} - \frac{2(2x-4)}{2x-4} = \frac{1}{x-2}$$

$$\frac{-3x+8}{2x-4} = \frac{1}{x-2}$$

$$(-3x+8)(x-2) = 2x-4$$

$$-3x^2+14x-16 = 2x-4$$

$$3x^2-12x+12 = 0$$

$$(3x-6)(x-2) = 0$$

$$x = 2$$

$\because x \neq 2, \therefore$ no solution

41. $$\frac{1}{x} = \frac{4}{3x} + 1$$

$$3 = 4 + 3x$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

42. $$\frac{2}{x} - 5 = \frac{6}{x} + 4$$

$$2 - 5x = 6 + 4x$$

$$9x = -4$$

$$x = -\frac{4}{9}$$

43. $\frac{2x-1}{x+2} = \frac{4}{5}$

$5(2x-1) = 4(x+2)$

$10x - 5 = 4x + 8$

$6x = 13$

$x = \frac{13}{6}$

44. $\frac{2x-7}{2x+4} = \frac{2}{3}$

$3(2x-7) = 2(2x+4)$

$6x - 21 = 4x + 8$

$2x = 8 + 21$

$2x = 29$

$x = \frac{29}{2}$

48. $\dfrac{12x-5}{6x+3} = 2 - \dfrac{5}{x}$

Method 1:

$$\frac{12x-5}{6x+3} = \frac{2x}{x} - \frac{5}{x}$$

$$\frac{12x-5}{6x+3} = \frac{2x-5}{x}$$

$$(12x-5)x = (2x-5)(6x+3)$$

$$12x^2-5x = 12x^2-24x-15$$

$$19x = -15$$

$$x = -\frac{15}{19}$$

Method 2:

$$12x^2-5x = 2x(6x+3)-5(6x+3)$$

$$12x^2-5x = 12x^2+6x-30x-15$$

$$12x^2-5x = 12x^2-24x-15$$

equivalent

Chapter 3 Polynomials and Rational Functions

1. Quadratic Functions and Models
2. Polynomial Functions and Their Graphs
3. Dividing Polynomials
4. Real Zeros of Polynomials
5. Complex Zeros and the Fundamental Theorem of Algebra
6. Rational Functions
7. Polynomial and Rational Inequalities

15. $f(x) = x^2 + 4x + 3$

(a) $f(x) = x^2 + 4x + 3 + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2$

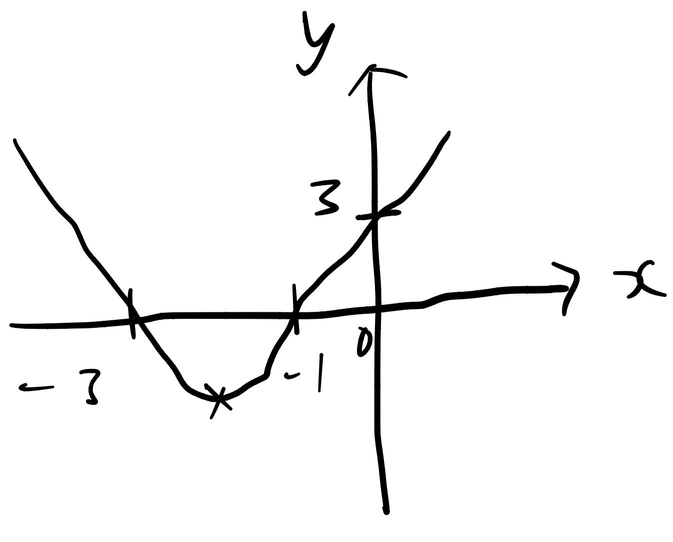
$= (x+2)^2 - 1$ $\quad (x+3)(x+1)$

(b) vertex = $(-2, -1)$

x-intercepts = $-3, -1$

y-intercepts = 3

(c)



$\boxed{2}$ Domain: $(-\infty, \infty)$

(d) Domain: $\{x \mid x \in R\}$

Range: $[-1, \infty)$

27. $f(x) = 3x^2 - 6x + 1$

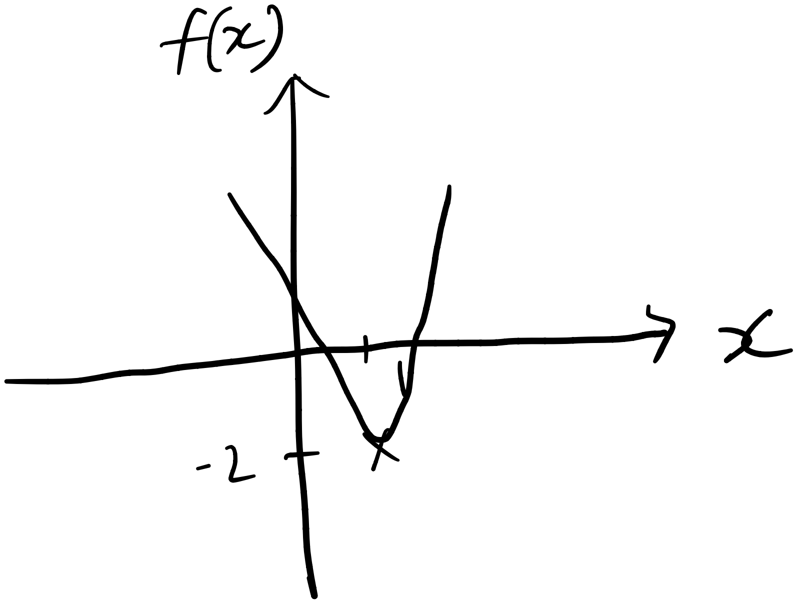
(a) $f(x) = 3\left(x^2 - 2x + \frac{1}{3}\right)$

$= 3\left(x^2 - 2x + \frac{1}{3} + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right)$

$= 3\left((x-1)^2 - \frac{2}{3}\right)$

$= 3(x-1)^2 - 2$

(b)



(c) minimum value: $y = -2$

3.1 Quadratic Functions and Models

① Graphing Quadratic Functions

② Maximum and Minimum Values

① Graphing Quadratic Functions

9. $f(x) = x^2 - 2x + 3$

(a) $f(x) = x^2 - 2x + 3$

$= x^2 - 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + 3$

$= (x-1)^2 + 2$

(b) Vertex: $(1, 2)$

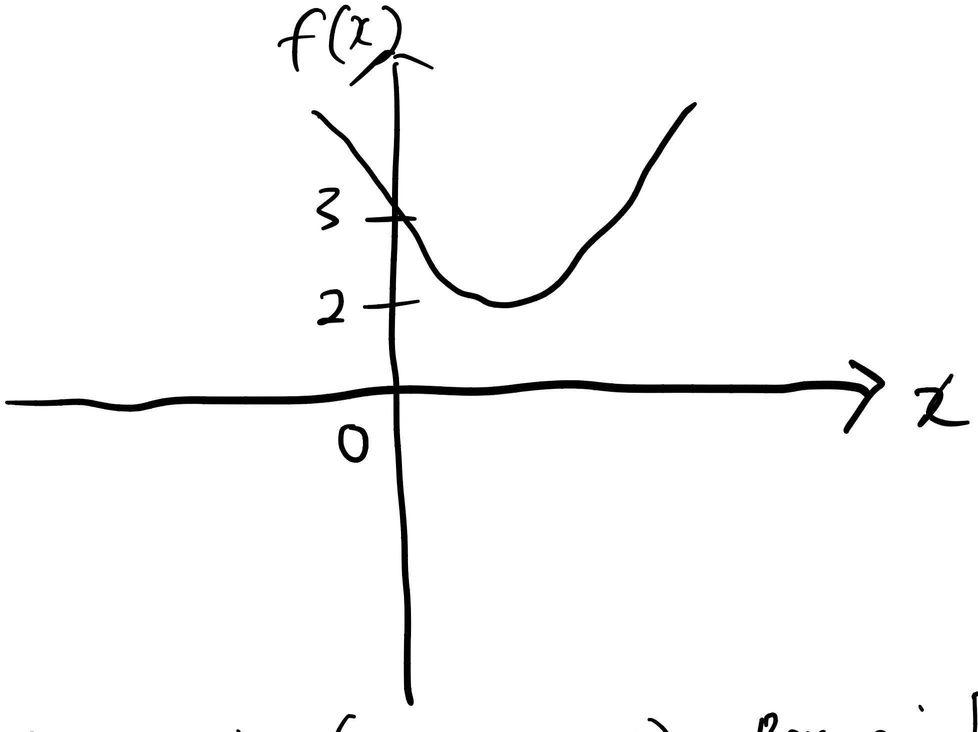
$b^2 - 4ac = 4 - 12$

$= -8 < 0$

$\therefore$ no x-intercept

y-intercept $= 1 + 2 = 3$

(c)



(d) Domain: $(-\infty, \infty)$, Range: $[2, \infty)$

10. $f(x) = x^2 + 4x - 1$

(a) $f(x) = x^2 + 4x - 1$

$= x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 1$

$= (x+2)^2 - 5$

(b) vertex : $(-2, -5)$

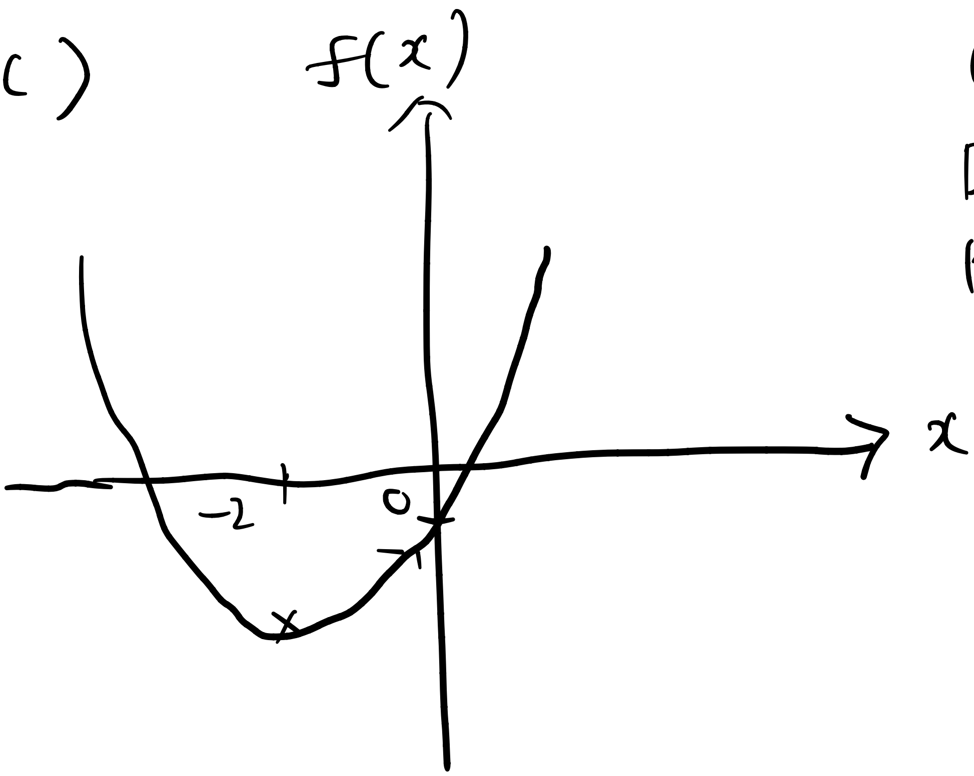
y-interept : -1

x-interrept : $x = \frac{-4 \pm \sqrt{16+4}}{2}$

$= \frac{-4 \pm 2\sqrt{5}}{2}$

$= -2 \pm \sqrt{5}$

(c)



(d)

Domain : $(-\infty, \infty)$

Range : $\{y \mid y \geq -5\}$

$[-5, \infty)$

11. $f(x) = x^2 - 6x$

(a) $f(x) = x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2$

$= (x-3)^2 - 9$

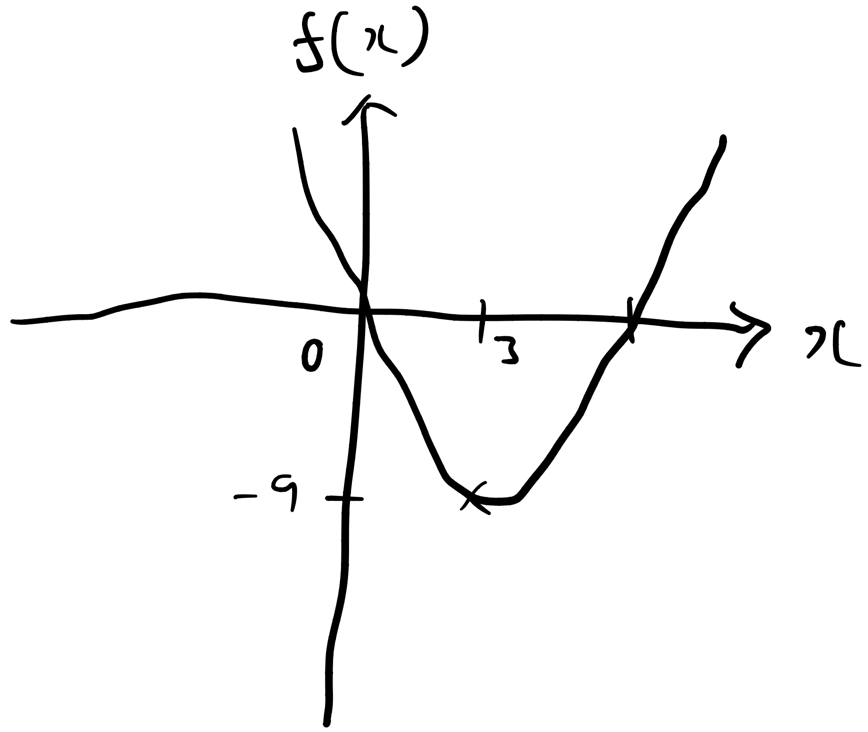
(b) vertex: $(3, -9)$

x-intercept: $x(x-6) = 0$

$x = 0, 6$

y-intercept: $y = 0$

(c)



(d) Domain: $\{x \mid x \in R\}$

Range: $[-9, \infty)$

12. $f(x) = x^2 + 8x$

(a) $f(x) = x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2$

$= (x+4)^2 - 16$

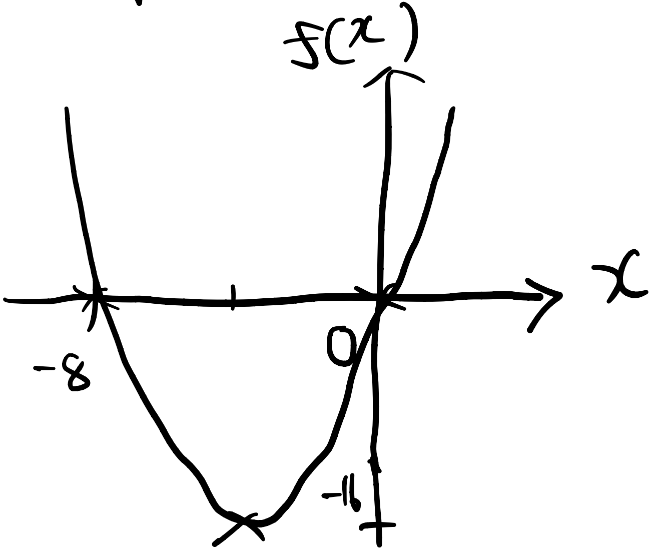
(b) Vertex: $(-4, -16)$

x-intercept: $x(x+8) = 0$

$x = 0, -8$

y-intercept: $y = 0$

(c)



(d) Domain: $(-\infty, \infty)$

Range: $[-16, \infty)$

13. $f(x) = 3x^2 + 6x$

(a) $f(x) = 3(x^2 + 2x)$

$= 3\left(x + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2\right)$

$= 3(x+1)^2 - 3$

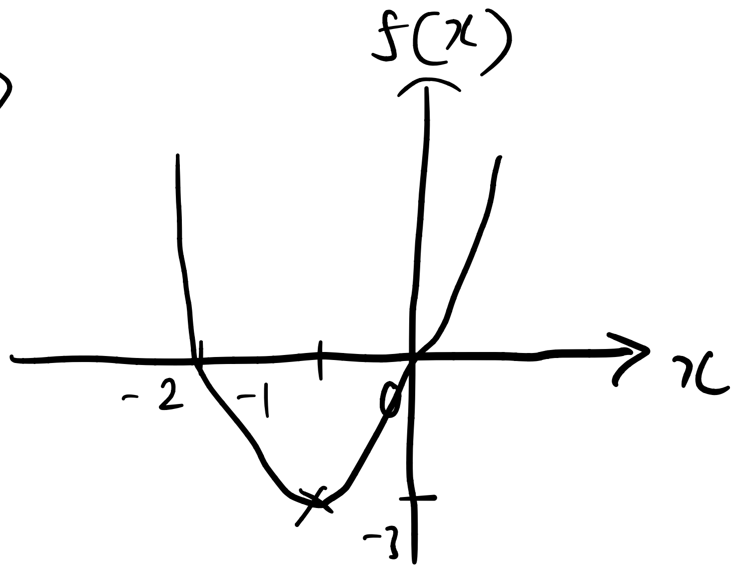
(b) Vertex : $(-1, -3)$

x-intercept : $3x(x+2) = 0$

$x = 0, -2$

y-intercept : $f(0) = 0$

(c)



(d) Domain : $(-\infty, \infty)$

Range : $[-3, \infty)$

14. $f(x) = -x^2 + 10x$

(a) $f(x) = -\left(x^2 - 10x + \left(\frac{10}{2}\right)^2 - \left(\frac{10}{2}\right)^2\right)$

$= -(x-5)^2 + 25$

(b) V : $(5, 25)$

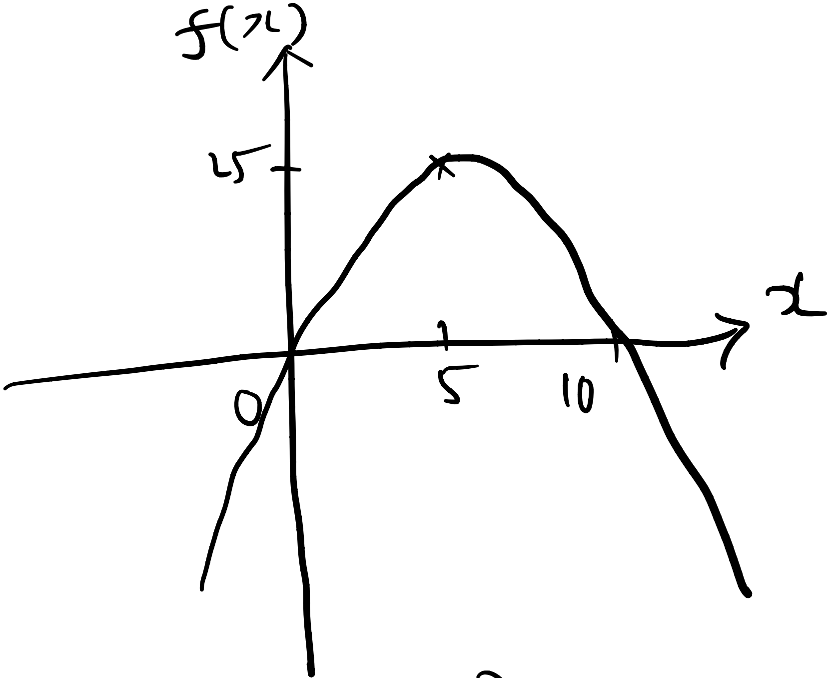
x-intercept : $-x^2 + 10x = 0$

$-x(x-10) = 0$

$x = 0, 10$

y-intercept : $f(0) = 0$

(c)



(d) Domain : $(-\infty, \infty)$

Range : $(-\infty, 25]$

15. $f(x) = x^2 + 4x + 3$

(a) $f(x) = x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 + 3$

$= (x+2)^2 - 1$

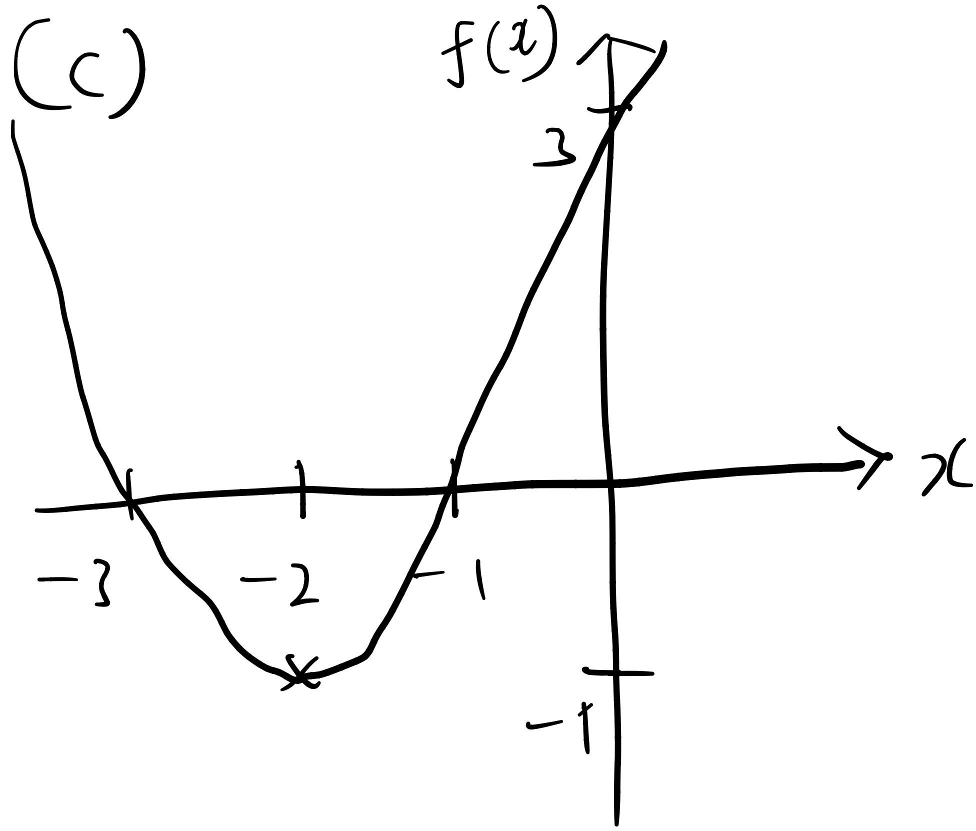
(b) Vertex : $(-2, -1)$

x-intercept : $f(x) = (x+3)(x+1)$

$x = -3, -1$

y-intercept : $f(0) = 3$

(c)



(d) Domain = $(-\infty, \infty)$

Range = $[-1, \infty)$

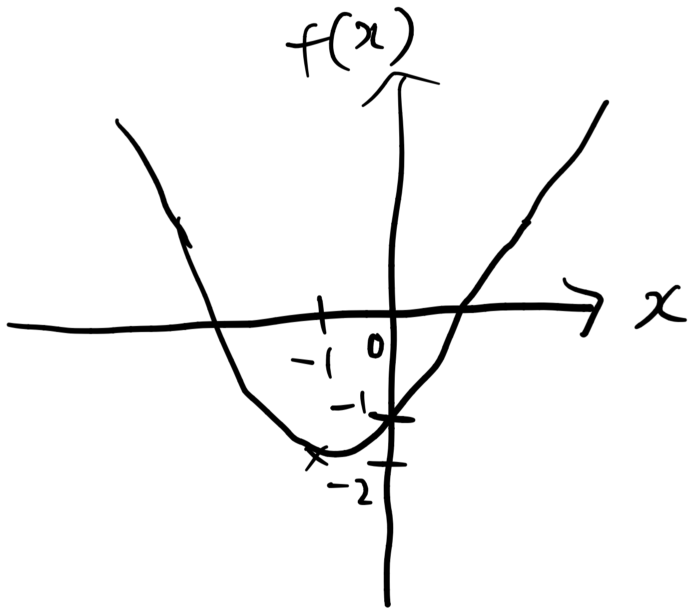
② Maximum and Minimum values

25. $f(x) = x^2 + 2x - 1$

(a) $x^2 + 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 - 1$

$= (x+1)^2 - 2$

(b)



(c) minimum value: $y = -2$

15/3

26. $f(x) = x^2 - 8x + 8$

(a) $f(x) = x^2 - 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 8$

$= (x-4)^2 - 8$

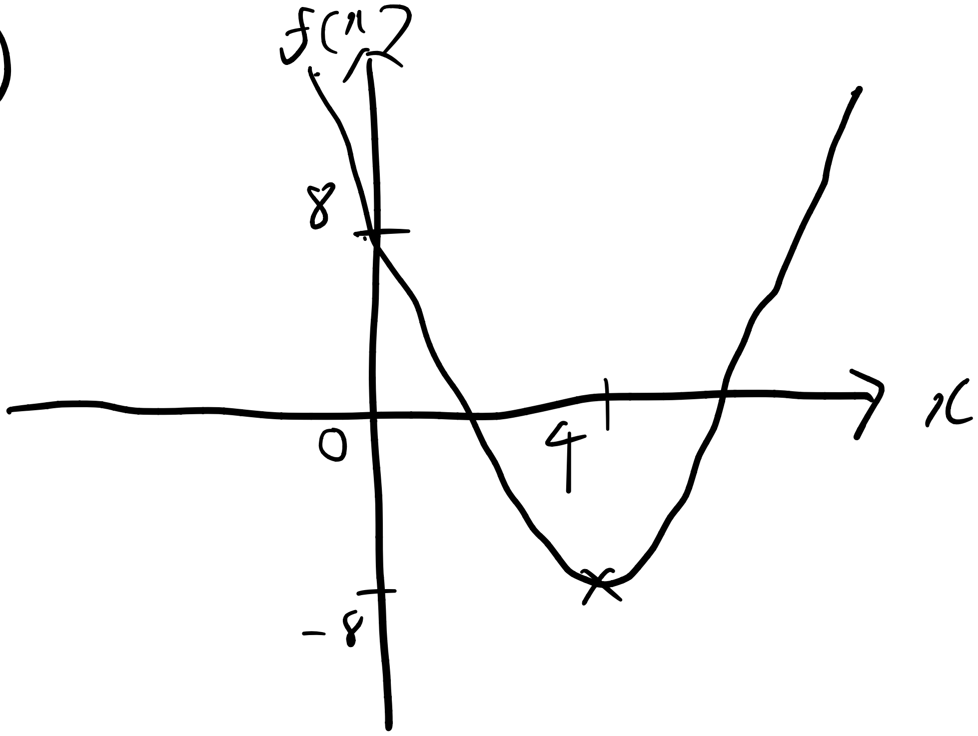
x-intercept:

$x = \dfrac{8 \pm \sqrt{64 - 32}}{2}$

$= \dfrac{8 \pm 4\sqrt{2}}{2}$

$= 4 \pm 2\sqrt{2}$

(b)



(c) Minimum value: $y = -8$

27. $f(x) = 3x^2 - 6x + 1$

$3x^2 - 6x + 1 = 0$

$x = \dfrac{6 \pm \sqrt{36 - 12}}{6}$

$= \dfrac{6 \pm \sqrt{24}}{6}$

$= \dfrac{6 \pm 2\sqrt{6}}{6}$

$= \dfrac{3 \pm \sqrt{6}}{3}$

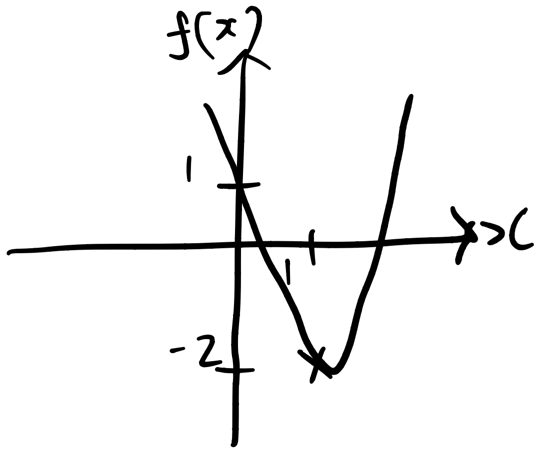
(a) $f(x) = 3\left(x^2 - 2x + \frac{1}{3}\right)$

$= 3\left(x^2 - 2x + \left(\frac{2}{2}\right)^2 - \left(\frac{2}{2}\right)^2 + \frac{1}{3}\right)$

$= 3\left((x-1)^2 - \frac{2}{3}\right)$

$= 3(x-1)^2 - 2$

(b)



(c) Minimum value:

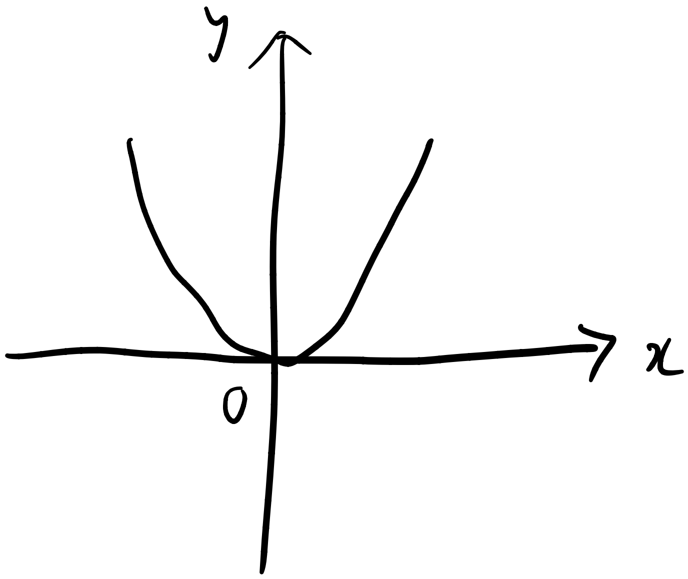
$y = -2$

3.2 Polynomial Functions and Their Graphs

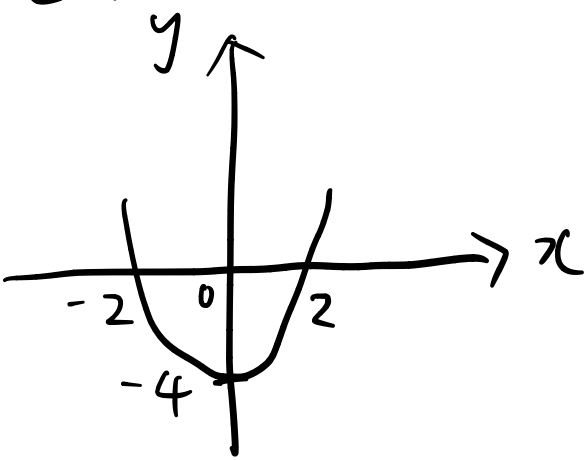
① Graphing Polynomials

② Local Extrema

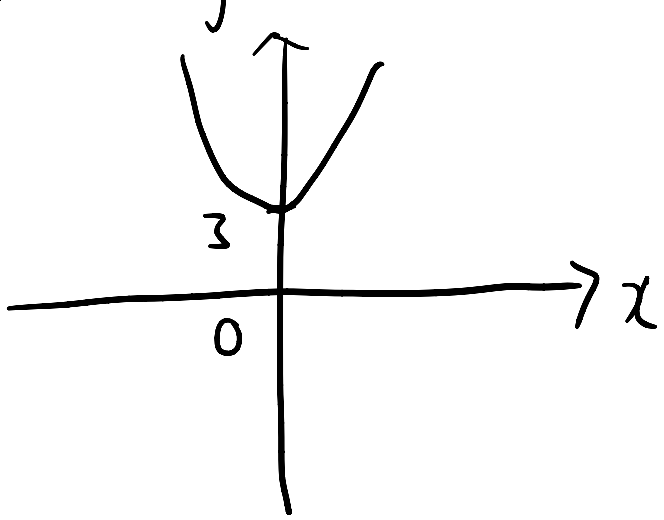
5. $y = x^2$



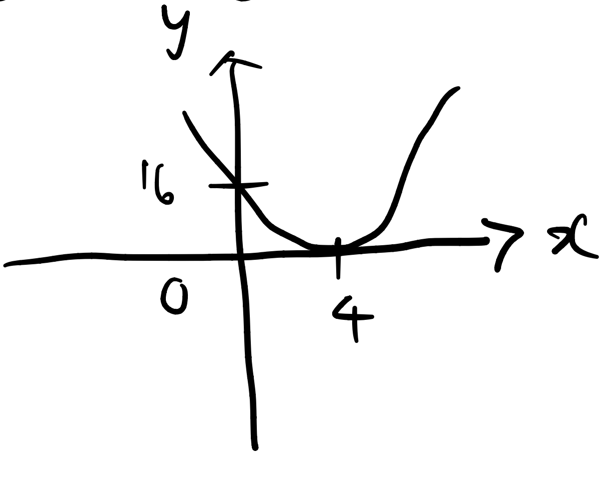
(a) $P(x) = x^2 - 4$



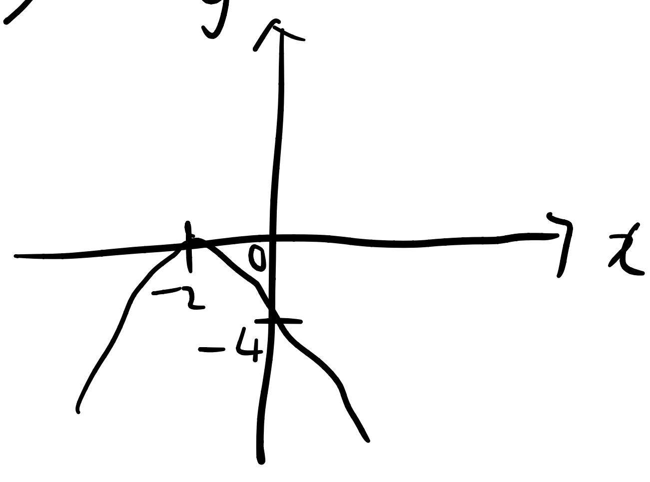
(c) $P(x) = 2x^2 + 3$



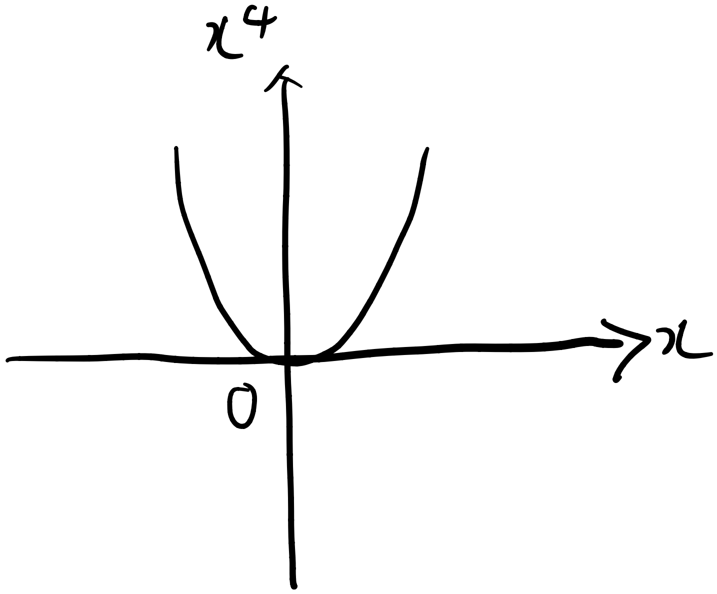
(b) $Q(x) = (x-4)^2$



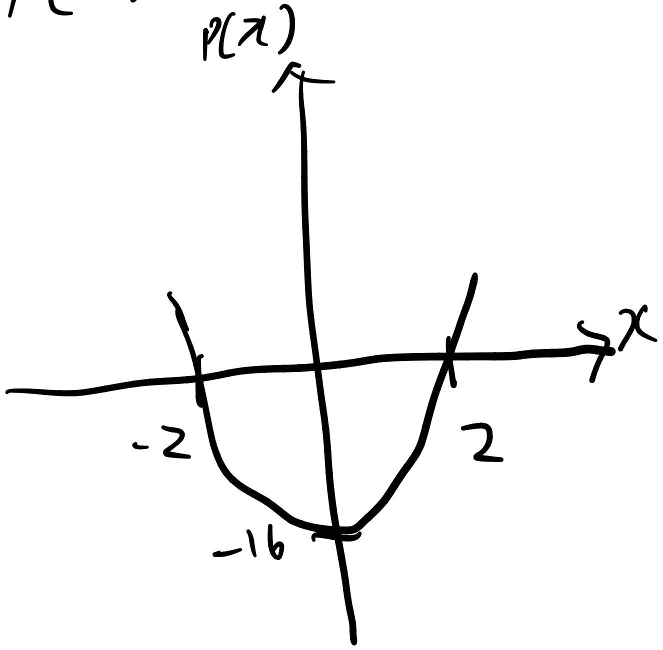
(d) $P(x) = -(x+2)^2$



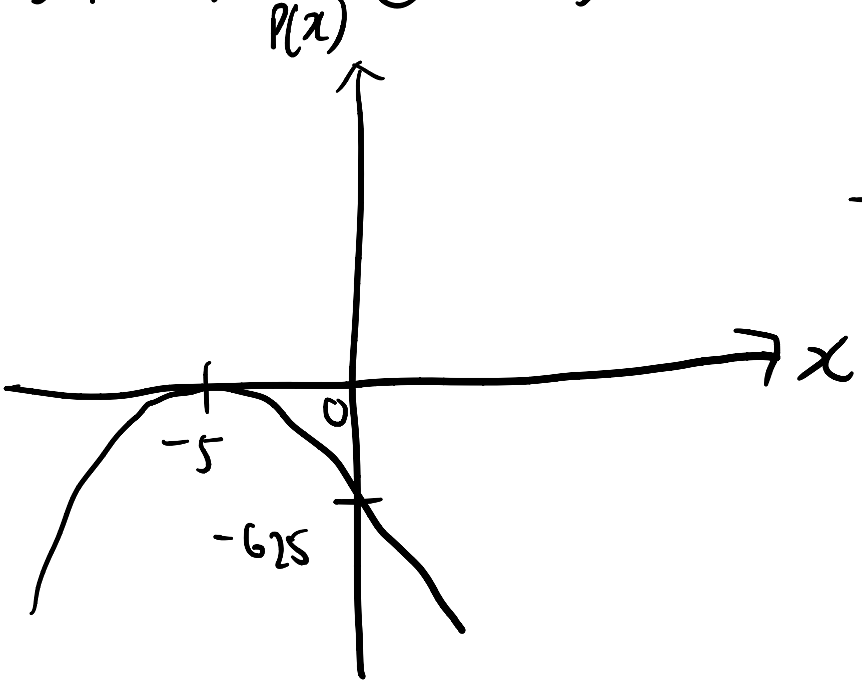
6.



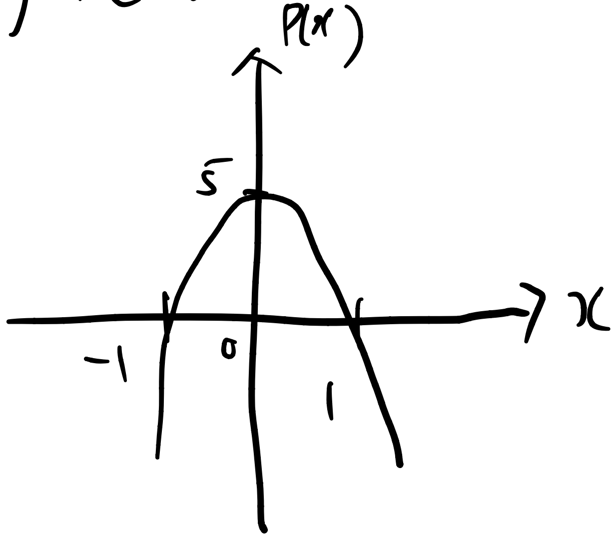
(a) $P(x) = x^4 - 16$



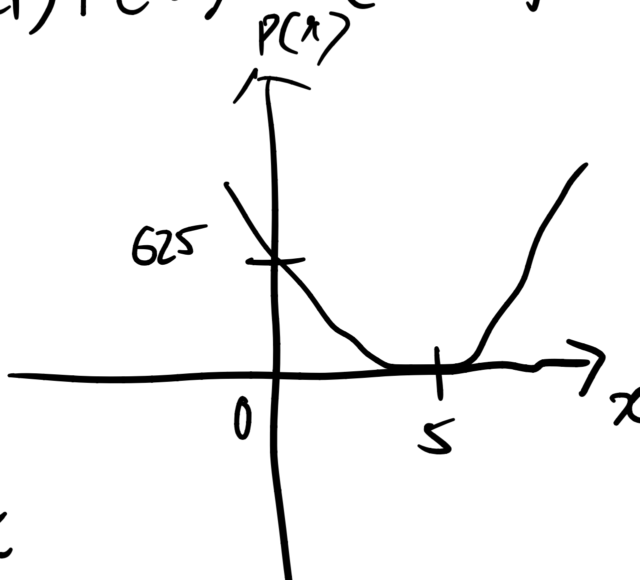
(b) $P(x) = -(x+5)^4$



(c) $P(x) = -5x^4 + 5$



(d) $P(x) = (x-5)^4$



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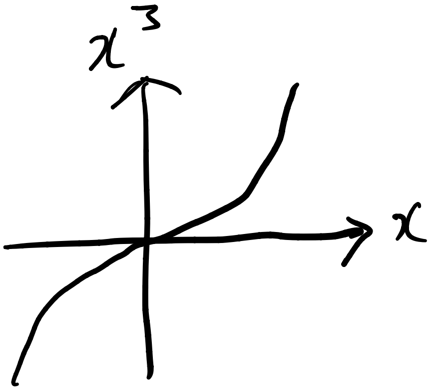
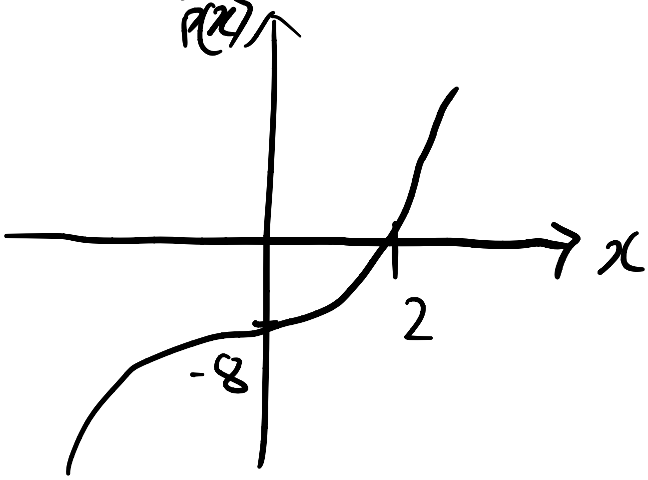
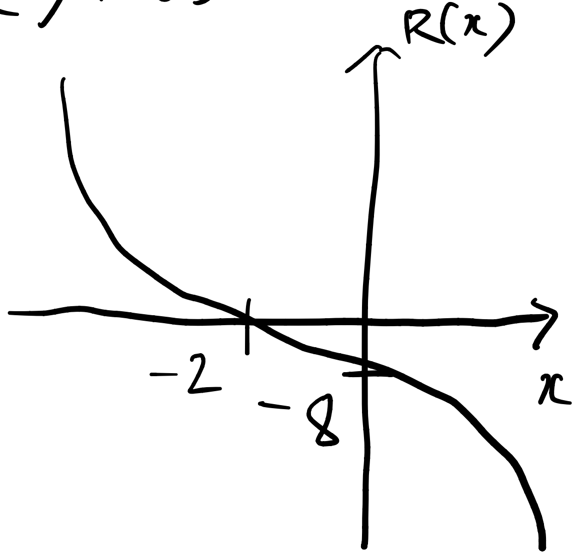
7. x^3



(a) $P(x) = x^3 - 8$

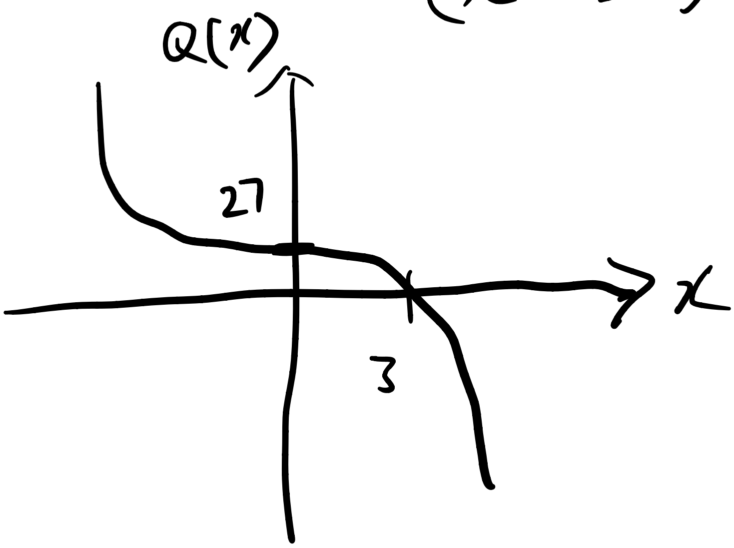


(c) $R(x) = -(x+2)^3$

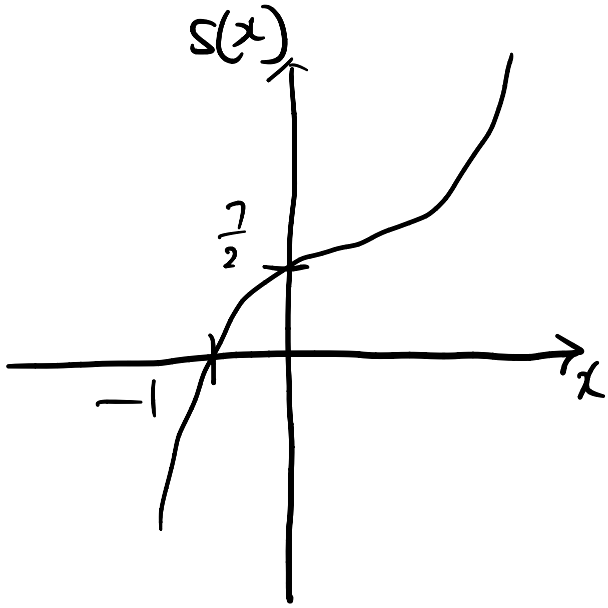


(b) $Q(x) = -x^3 + 27$

$= -(x^3 - 27)$

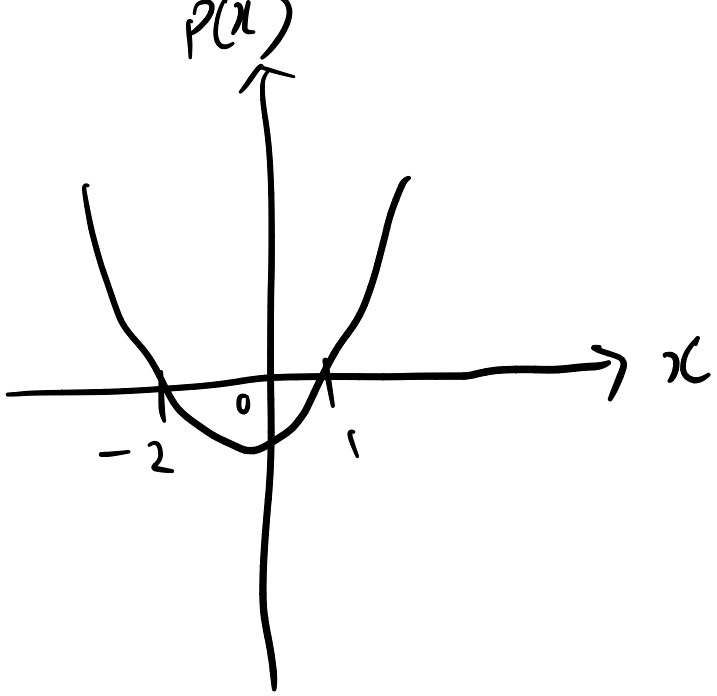


(d) $S(x) = \frac{1}{2}(x-1)^3 + 4$



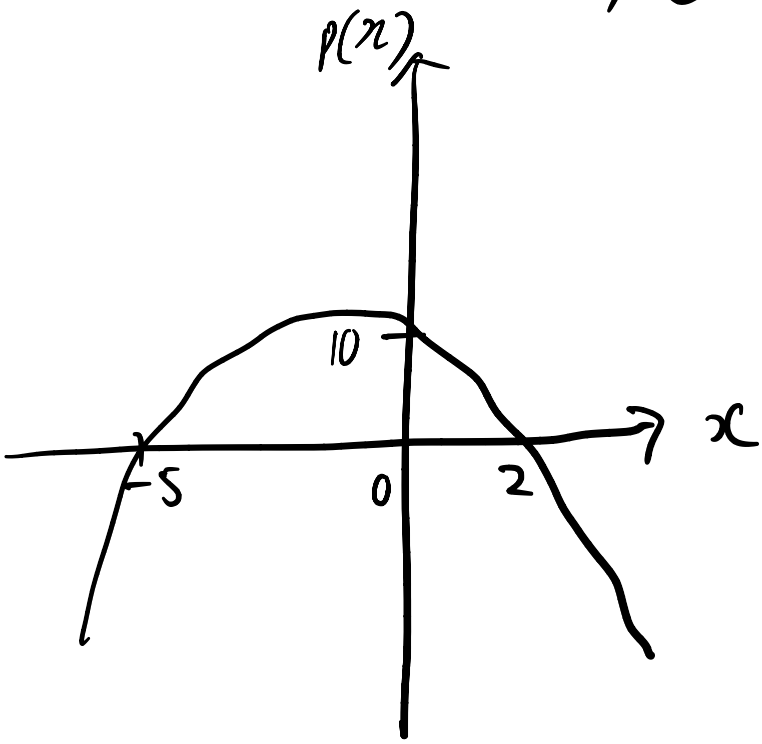
① Graphing Polynomials

15. $P(x) = (x-1)(x+2)$

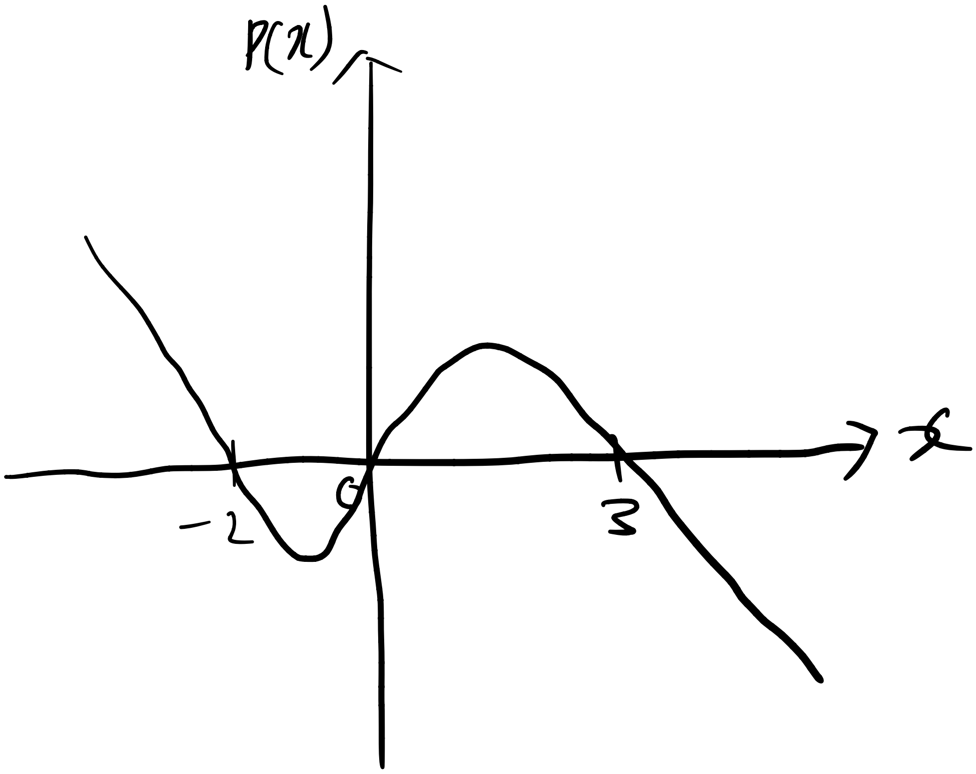


16. $P(x) = (2-x)(x+5)$

$= -(x-2)(x+5)$

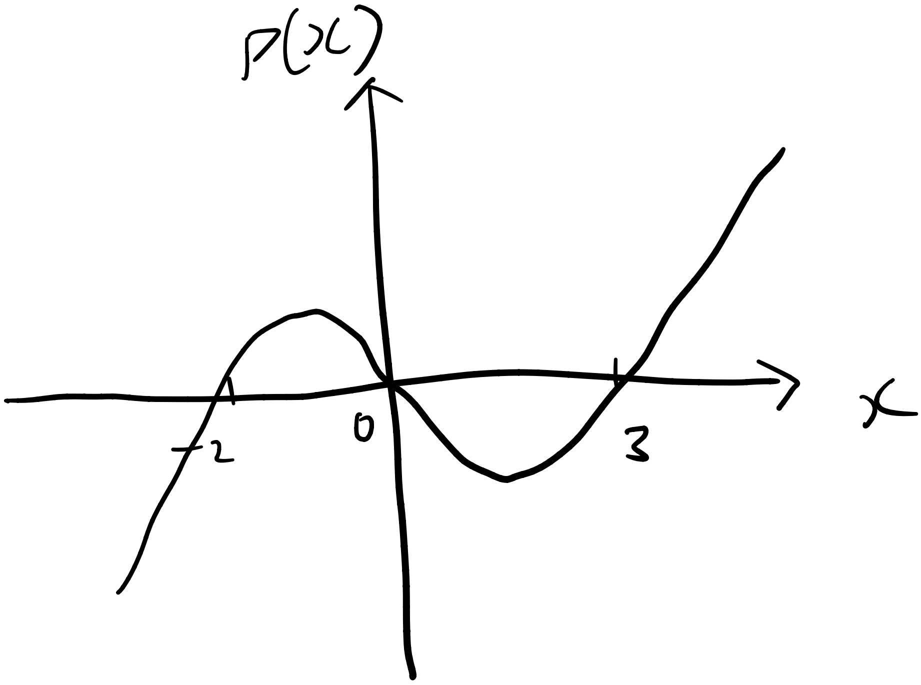


17. $p(x) = -x(x-3)(x+2)$



18. $p(x) = x(x-3)(x+2)$

$x = -2, 0, 3$



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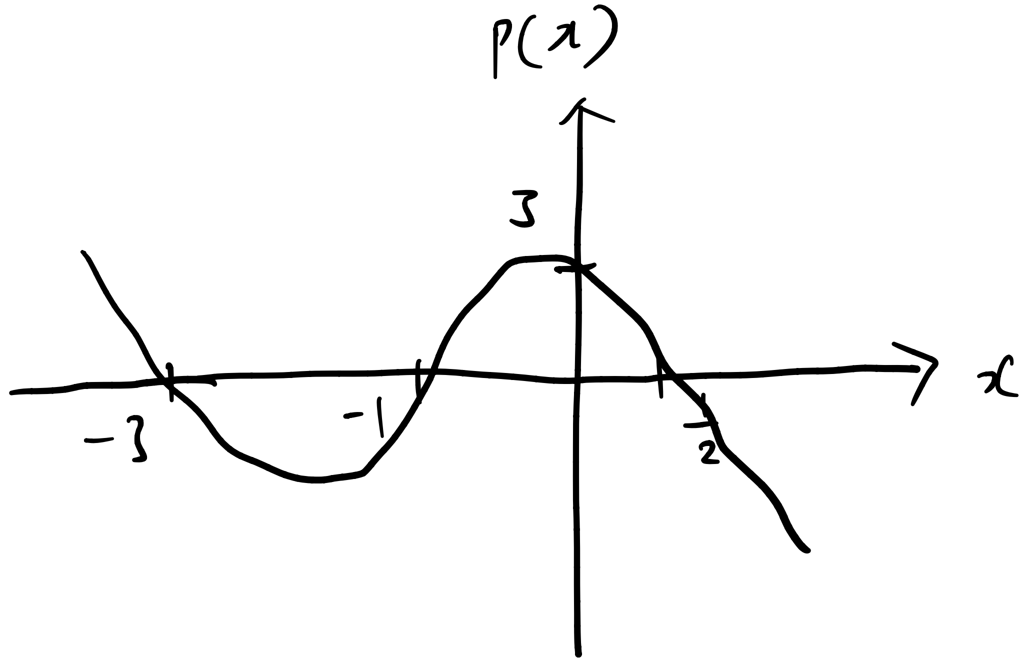
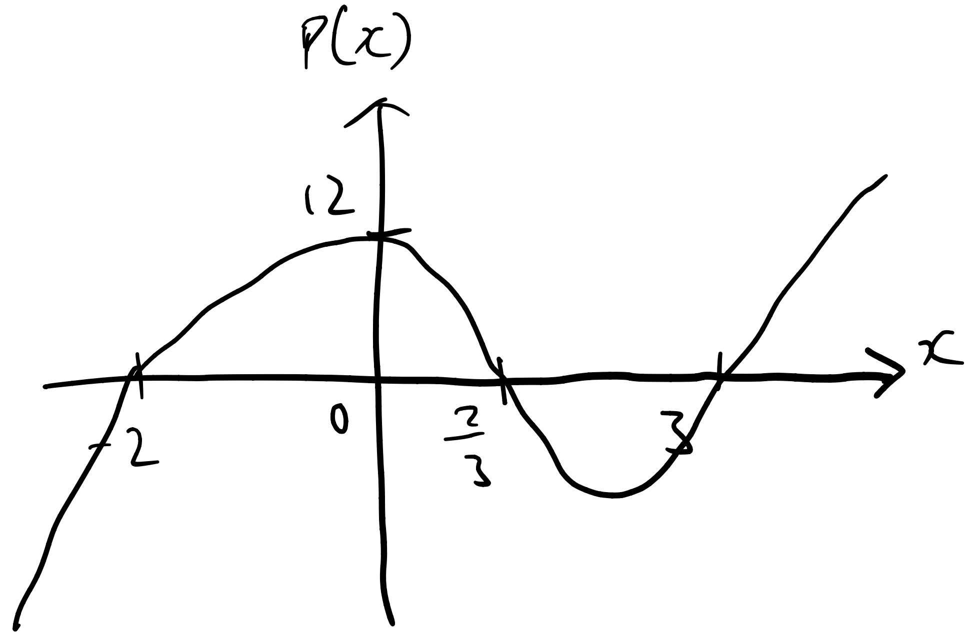
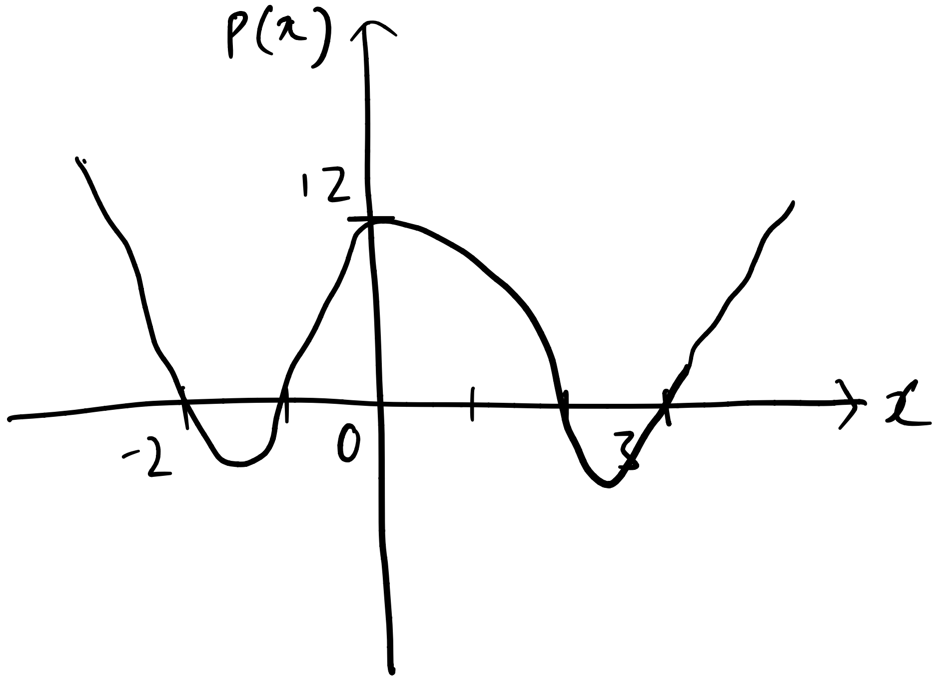
19. $P(x) = -(2x-1)(x+1)(x+3)$



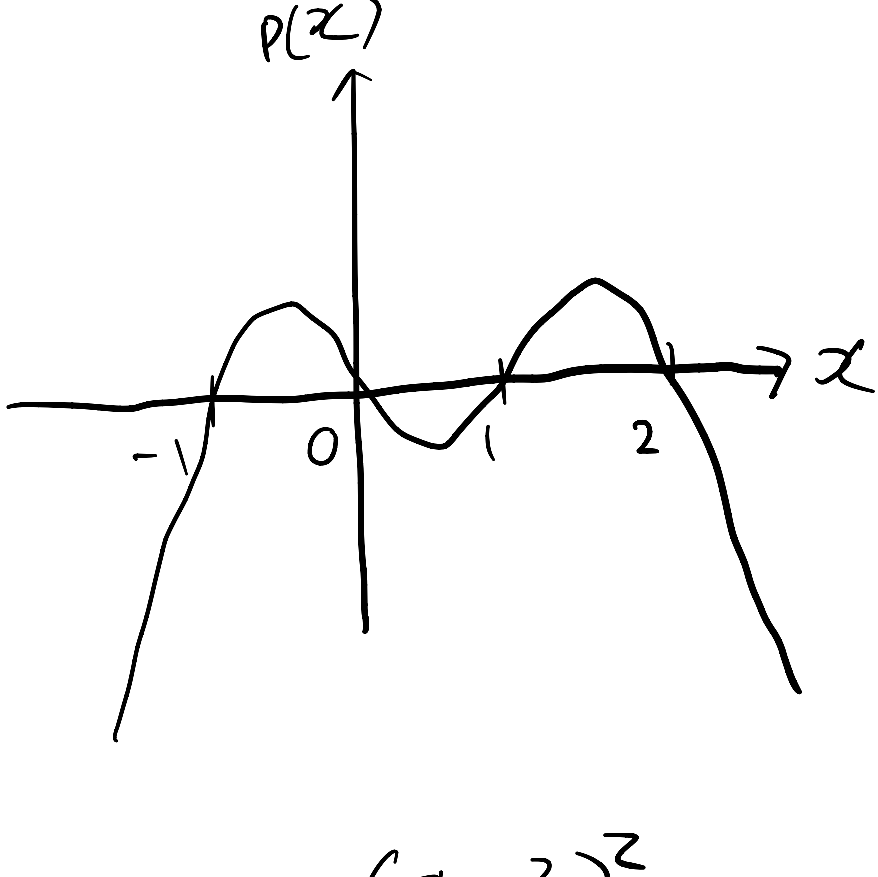
20. $P(x) = (x-3)(x+2)(3x-2)$



21. $P(x) = (x+2)(x+1)(x-2)(x-3)$

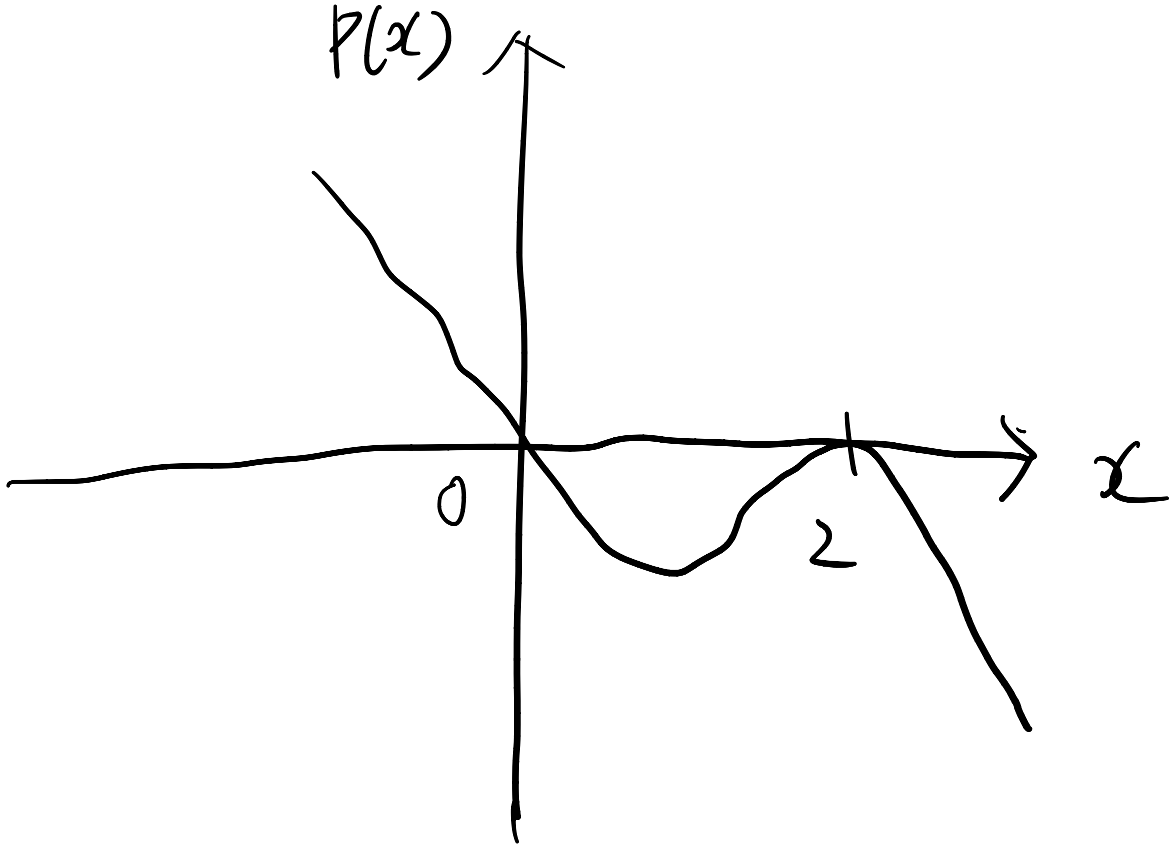


22. $P(x) = x(x+1)(x-1)(2-x) = -x(x+1)(x-1)(x-2)$



23. $P(x) = -2x(x-2)^2$

$x = 0, 2$


P(x)
x
0
2

② Local Extrema

51. $P(x) = -x^2 + 4x$

(b) $(2, 4)$

(a) $y = 0$

$x = 0, 4$

52. $P(x) = \frac{2}{9}x^3 - x^2$

$= -x^2\left(1 - \frac{2}{9}x\right)$

(a) $y = 0$

$x = 0, 4.5$

(b) $(0, 0), (3, -3)$

53. $P(x) = -\frac{1}{2}x^3 + \frac{3}{2}x - 1$

$= -\frac{1}{2}(x^3 - 3x + 2)$

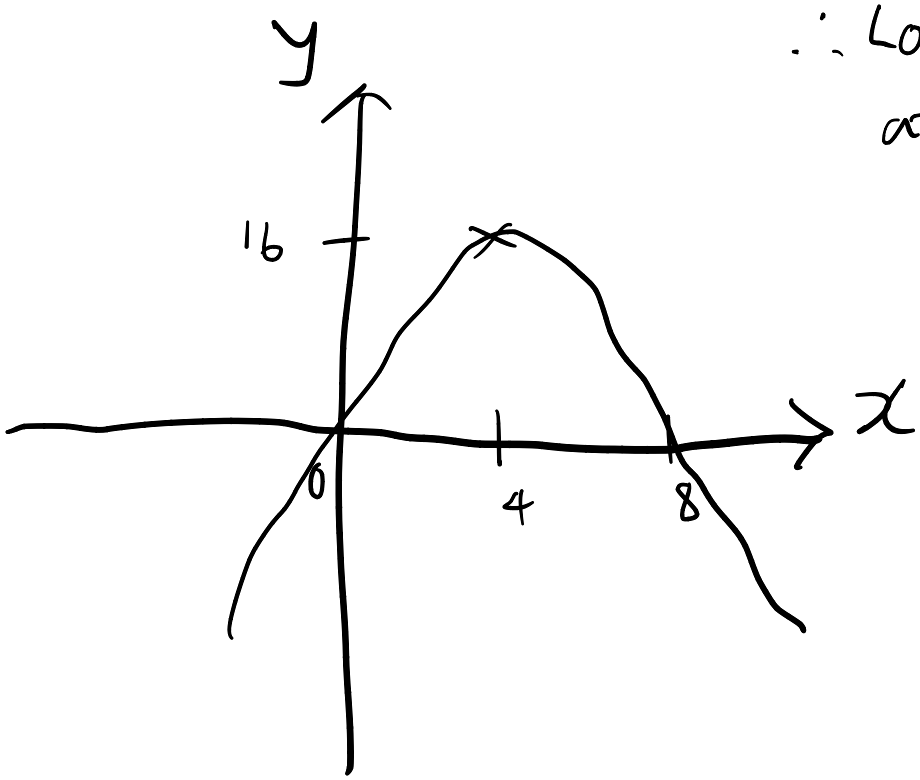
(a) $x = -2, 1$

$y = -1$

(b) $(-1, -2), (1, 0)$

55. $y = -x^2 + 8x$, $[-4, 12]$ by $[-50, 30]$

$$y = -x^2 + 8x$$
$$= -(x^2 - 8x)$$
$$= -\left(x^2 - 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2\right)$$
$$= -\left((x-4)^2 - 16\right)$$
$$= -(x-4)^2 + 16$$



$\therefore$ Local maximum at $(4, 16)$

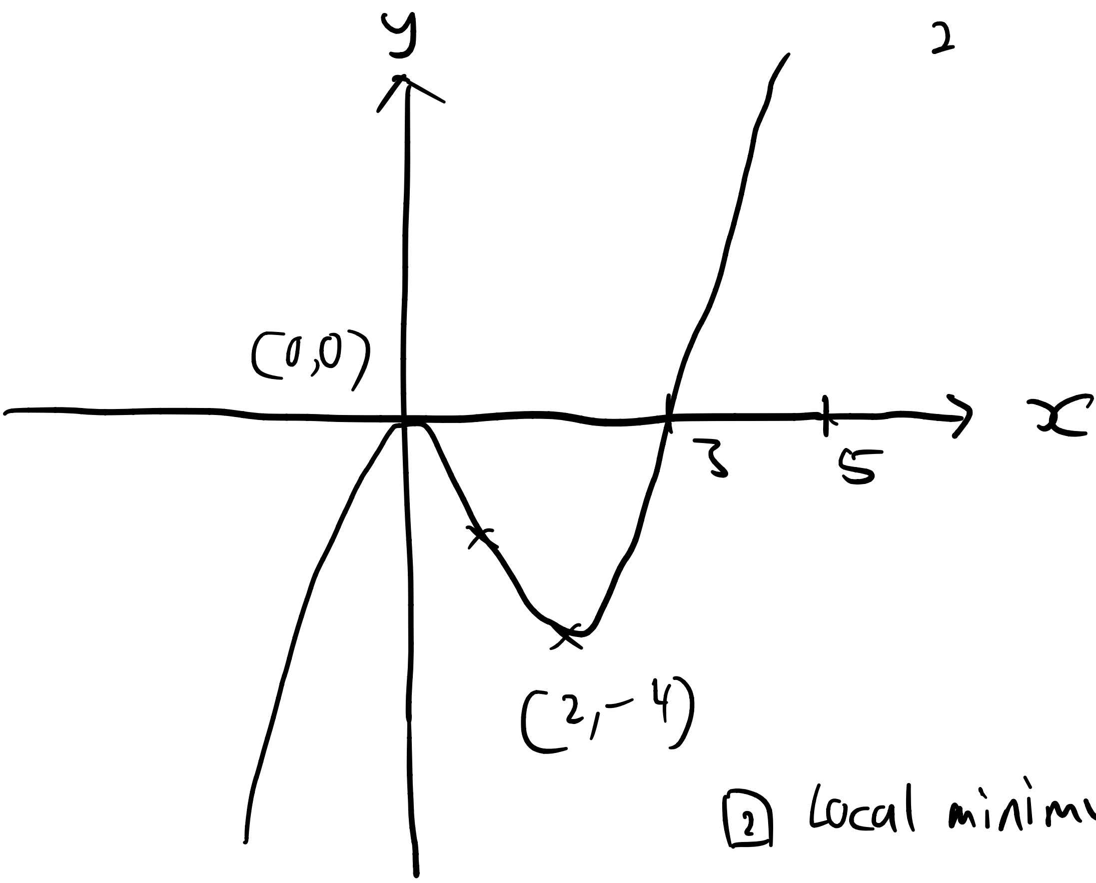
Domain: $(-\infty, \infty)$, Range: $(-\infty, 16]$

56. $y = x^3 - 3x^2$, $[-2, 5]$ by $[-10, 10]$

$y = x^2(x - 3) = 0$

$x = 0, 3$

x	y
1	-2
2	-4



$\boxed{2}$ Local minimum at $(2, -4)$

Local maximum at $(0, 0)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

3.3 Dividing Polynomials

1. Division of Polynomials
2. Remainder Theorem
3. Factor Theorem

① Division of Polynomials

3. $P(x) = 2x^2 - 5x - 7, \quad D(x) = x - 2$

$$
\begin{array}{r|l}
 & 2x - 1 \\
\hline
x-2 & 2x^2 - 5x - 7 \\
 & 2x^2 - 4x \\
\hline
 & \quad -x - 7 \\
 & \quad -x + 2 \\
\hline
 & \qquad\quad -9
\end{array}
$$

$$\frac{P(x)}{D(x)} = 2x - 1 + \frac{-9}{x-2}$$

4. $P(x) = 3x^3 + 9x^2 - 5x - 1, \quad D(x) = x + 4$

$$
\begin{array}{r|l}
 & 3x^2 - 3x + 7 \\
\hline
x+4 & 3x^3 + 9x^2 - 5x - 1 \\
 & 3x^3 + 12x^2 \\
\hline
 & \quad -3x^2 - 5x - 1 \\
 & \quad -3x^2 - 12x \\
\hline
 & \qquad\quad 7x - 1 \\
 & \qquad\quad 7x + 28 \\
\hline
 & \qquad\qquad -29
\end{array}
$$

$$\frac{P(x)}{D(x)} = 3x^2 - 3x + 7 \frac{-29}{x+4}$$

5. $P(x) = 4x^2 - 3x - 7$, $D(x) = 2x - 1$

$$\begin{array}{r|l} & 2x - \frac{1}{2} \\ \hline 2x-1 & 4x^2 - 3x - 7 \\ & \underline{4x^2 - 2x} \\ & \quad -x - 7 \\ & \quad \underline{-x + \frac{1}{2}} \\ & \qquad -7\frac{1}{2} \end{array}$$

$$\frac{P(x)}{D(x)} = 2x - \frac{1}{2} + \frac{-\frac{15}{2}}{2x-1}$$

$$= 2x - \frac{1}{2} - \frac{15}{4x-2}$$

6. $P(x) = 6x^3 + x^2 - 12x + 5$, $D(x) = 3x - 4$

$$\begin{array}{c|cccc} \frac{4}{3} & 6 & 1 & -12 & 5 \\ & & 8 & 12 & 0 \\ \hline & 6 & 9 & 0 & 5 \end{array}$$

$$\frac{P(x)}{D(x)} = 2x^2 + 3x + \frac{5}{3x-4}$$

7. $P(x) = 2x^4 - x^3 + 9x^2, \quad D(x) = x^2 + 4$

$$
\begin{array}{r l}
 & 2x^2 - x + 1 \\
x^2 + 4 \,\big) & 2x^4 - x^3 + 9x^2 \\
 & 2x^4 + 8x^2 \\ \hline
 & -x^3 + x^2 \\
 & -x^3 - 4x \\ \hline
 & x^2 + 4x \\
 & x^2 + 4 \\ \hline
 & 4x - 4
\end{array}
$$

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$

$$= 2x^2 - x + 1 + \frac{4x - 4}{x^2 + 4}$$

8. $P(x) = 2x^5 + x^3 - 2x^2 + 3x - 5, \quad D(x) = x^2 - 3x + 1$

$$
\begin{array}{r l}
 & 2x^3 + 6x^2 + 17x + 43 \\
x^2 - 3x + 1 \,\big) & 2x^5 + 0 + x^3 - 2x^2 + 3x - 5 \\
 & 2x^5 - 6x^4 + 2x^3 \\ \hline
 & 6x^4 - x^3 - 2x^2 \\
 & 6x^4 - 18x^3 + 6x^2
\end{array}
$$

$$17x^3 - 8x^2 + 3x$$
$$\underline{17x^3 - 51x^2 + 17x}$$
$$43x^2 - 14x - 5$$
$$\underline{43x^2 - 129x + 43}$$
$$115x - 48$$

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$$
$$= 2x^3 + 6x^2 + 17x + 43 + \frac{115x - 48}{x^2 - 3x + 1}$$

9. $P(x) = -x^3 - 2x + 6$, $D(x) = x + 1$

$$\begin{array}{r}
-x^2 + x - 3 \\
x+1 \,\big)\!\overline{-x^3 + 0 - 2x + 6} \\
\underline{-x^3 - x^2 \qquad\qquad} \\
x^2 - 2x \qquad \\
\underline{x^2 + x \qquad} \\
-3x + 6 \\
\underline{-3x - 3} \\
9
\end{array}$$

$$P(x) = D(x) \cdot Q(x) + R(x)$$
$$= (x+1)(-x^2 + x - 3) + 9$$

(2) Remainder Theorem

39. $p(x) = 4x^2+12x+5$, $c=-1$

$$\begin{array}{r|rrr} -1 & 4 & 12 & 5 \\ & & -4 & -8 \\ \hline & 4 & 8 & -3 \end{array}$$

$p(-1) = -3$

40. $p(x) = 2x^2+9x+1$, $c=\frac{1}{2}$

$$\begin{array}{r|rrr} \frac{1}{2} & 2 & 9 & 1 \\ & & 1 & 5 \\ \hline & 2 & 10 & 6 \end{array}$$

$$\begin{aligned} p\left(\tfrac{1}{2}\right) &= 2\left(\tfrac{1}{2}\right)^2+9\left(\tfrac{1}{2}\right)+1 \\ &= \frac{1}{2}+\frac{9}{2}+1 \\ &= 6 \end{aligned}$$

41. $P(x) = x^3 + 3x^2 - 7x + 6$, $c = 2$

$$
\begin{array}{r|rrrr}
2 & 1 & 3 & -7 & 6 \\
 & & 2 & 10 & 6 \\
\hline
 & 1 & 5 & 3 & 12
\end{array}
$$

$$
\begin{aligned}
P(2) &= (2)^3 + 3(2)^2 - 7(2) + 6 \\
&= 8 + 12 - 14 + 6 \\
&= 12
\end{aligned}
$$

③ Factor Theorem

53. $P(x) = x^3 - 3x^2 + 3x - 1$, $c = 1$

$$P(1) = 1^3 - 3(1)^2 + 3(1) - 1$$
$$= 1 - 3 + 3 - 1$$
$$= 0$$

Factor theorem: $P(1) = 0$, $c = 1$ is a factor of $P(x)$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$$P(x) = (x-1)(x^2 - 2x + 1)$$

54. $P(x) = x^3 + 2x^2 - 3x - 10$, $c = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -3 & -10 \\ & & 2 & 8 & 10 \\ \hline & 1 & 4 & 5 & 0 \end{array}$$

$$P(2) = 8 + 8 - 6 - 10$$
$$= 0$$

$$P(x) = (x-2)(x^2 + 4x + 5)$$